UWB Channel Estimation Based on Distributed Bayesian Compressive Sensing

Tang Liang, Zhou Zheng, Shi Lei

Key Lab of Universal Wireless Communications, MOE
Wireless Network Lab, Beijing University of Posts and Telecommunication
E-mail: tangliangbupt@gmail.com

Abstract

In order to solve the high sampling rate issue in the multiuser UWB communication network system, we process the received signal with Bayesian compressive sensing. Using the characteristic that the wireless channels of multiuser signals which are received by one receiver at the same time are statistically related, a Laplace prior based distributed Bayesian compressive sensing method is proposed. It jointly reconstructs the received signals from different users and gets the parameters of channel models. The proposed method reduces the necessary sampling numbers for channel estimation. The experiment shows that the method improves the BER performance.

Keywords: Distributed Bayesian Compressive Sensing, Laplace Prior, Channel Estimation

1. Introduction

UWB wireless communication technology is a hot issue in the communication research area in recent years. The high transmission rate, low transmission power and anti-jamming capability make it become a short-range wireless communication technology with good prospect. However, the high sampling rate limits the practical application of UWB wireless communication technology.

Recently, Compressive Sensing (CS) theory [1-4] has been applied to UWB [5] system to solve the problem of high sampling rate. Compressive Sensing theory uses a special signal reconstruction method to make the signal be accurately reconstructed with the much less samples than the number required by Nyquist criterion. Literature [6] sets filter in the transmitter to construct a compressive sensing system. The literature [7] proposes a method which carries out channel estimation by regarding the UWB multipath channels as the atoms of the dictionary.

The literatures only consider the situation that a single received signal is processed with Compressive Sensing algorithm. This paper considers that in the multiuser UWB communication system, receiver simultaneously receives signals from different transmitters. This paper proposes the distributed Bayesian Compressive Sensing (BCS) [8] algorithm to jointly reconstruct the received signals from different users and get the parameters of channel models. Contrast to the MT-BCS [9], the algorithm proposed in this paper utilizes Laplace prior to improve the reconstruction performance. Simulation results show that comparing with other compressive sensing algorithms, the proposed reconstruction algorithm reduces the number of measurements required by UWB channel estimation and improves the system performance.

The rest of this paper is organized as follow: in section II, we analyze the system model. Section III proposes a reconstruction algorithm. Section IV presents the experiment results and analyzes it. The conclusion is drawn in section V.

2. System Model

This paper uses the dictionary construction in literature [7], which is generated by shifting the wave function with minimum step $\Delta$. Taken $\Delta$ as the sampling period, this dictionary is a complete redundant dictionary. This dictionary structure well reflects the multipath feature of...
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UWB signal. Supposing the total number of atoms in the dictionary is N, the sparse representation of received UWB signal can be written as:

\[ x = \sum_{k=1}^{N} \alpha_k p(t - \tau_k \Delta) \]  

(1)

where \( \alpha_k \) is the attenuation coefficient of \( k \)th multipath channel, \( \tau_k \Delta \) is the delay parameter of \( k \)th atom. If there is no multipath component in the delay \( \tau_k \Delta \), the corresponding attenuation coefficient \( \alpha_k \) is zero. In the literature [7], the channel is time-invariant, so \( \alpha_k \) and \( \tau_k \Delta \) have fixed value. However, in this paper, attenuation coefficient \( \alpha_k \) and delay parameter \( \tau_k \Delta \) are assumed to meet the certain probability distribution. In the (1), \( p(t) \) is the UWB pulse waveform function and \( p(t - \tau_k \Delta) \) and \( \alpha_k \) are the sparse basis and sparse coefficient of the received signal respectively. As the different emission sources transmit signal in the same space at the same time, thus the channel models of different received signals are assumed to be same, namely, the channel parameters \( \alpha_k \) and \( \tau_k \Delta \) of different signals have the same probability distribution. Because non-zero \( \alpha_k \) satisfies some probability distribution in this paper, so unlike literature [7], Bayesian Compressive Sensing algorithm is used. From the above sparse signal model, we can get that because \( \alpha_k \) and \( \tau_k \Delta \) of different signals have the same probability distribution, the sparse coefficients of different signals received simultaneously have the same probability distribution. Thus, sparse coefficients of different signals are statistically related. According to the characteristics of sparse coefficient of different signals satisfying the same distribution, distributed Bayesian compressive sensing can be applied to dealing with the received signal group. Diagram is shown as follow:

![Figure 1. Block diagram of distributed Bayesian compressive sensing based UWB channel estimation](image)

In the Figure 1, we assume that the receiver receives the signal from \( L \) users at the same time. \( Y = [y_1, y_2, \ldots, y_L] \) is the measurement group of received signals through measuring and \( y_l \) is the measurement of \( l \)th received signal. The obtained \( L \) measurements are sent to the joint reconstruction device through \( L \) branches simultaneously to get the channel models \( h_1, h_2, \ldots, h_L \) of received signals. \( h_l \) is the channel model of \( l \)th received signal. The obtained channel parameters are used for the recovery of the signal by receiver. The independent compressive sensing reconstruction process in the former research is changed into the process of joint reconstruction for many signals.
3. Joint Reconstruction Algorithm

Unlike the linear programming \cite{10} and greedy algorithm \cite{11}, Bayesian Compressive Sensing (BCS) uses the prior probability distribution of original signal. BCS is based on the fast relevance vector machine (RVM) \cite{12}. Literature \cite{9} proposed a multi-task Bayesian Compressive Sensing reconstruction method (MT-BCS), but it is based on the Gaussian probability distribution of original signal. Literature \cite{13} shows that the Laplace prior based Bayesian Compressive Sensing algorithm outperforms Gaussian prior based Compressive Sensing algorithm. Therefore, in this paper, Laplace prior probability distribution is used in the distributed Bayesian Compressive Sensing algorithm.

The $L$ sets of measurements are represented as $\{y_i\}_{i=1}^{L}$, where $y_i = \mathbf{y}_i \mathbf{w}_{n} + \mathbf{x}_i \in \mathbb{R}^N$ is $i$th signal, and every measurement vector $y_i \in \mathbb{R}^{M_i}$ employs a different random projection matrix $\mathbf{\Phi}_i \in \mathbb{R}^{M_i \times N}$, where $M_i$ represents the number of measurements corresponding to $i$th signal. $\mathbf{n}_i$ is the measurement noise which is modeled as $\mathbf{n}_i \sim \text{i.i.d. draws of a zero-mean Gaussian random variable}$ with the precision $\beta$ (variance $1/\beta$). We put $\beta$ a fix value $\beta = 0.01^* \mathbf{v}_i \mathbf{v}_i$ like in \cite{8}.

For simplicity but without loss of generality, we assume that the sparse basis $\Psi = \mathbf{I}_N$, and we can obtain $\mathbf{x}_i = \Psi \theta_i = \theta_i$, then $y_i = \Phi \theta_i + n_i$. Thus the probability distribution of $y_i$ is

$$p(y_i | \theta_i, \beta) = \mathcal{N}(y_i | \Phi \theta_i, \beta^{-1})$$

(2)

The coefficients $\theta_i$ is assigned the Laplace prior with the parameter $\lambda$. This Laplace distribution is shared by all signals, and the $L$ signals are statistically related in this sense. That is

$$p(\theta_i | \lambda) = (\lambda / 2)^N \exp(-\lambda |\theta_i|)$$

(3)

However, this model of Laplace prior cannot be used directly, since it is not conjugate to the Gaussian distribution in (2). To solve this problem, the hierarchical prior is employed. Letting the $\theta_{i,j}$ represent the $j$th sparse coefficient of the $i$th signal, we first assign the zero-mean Gaussian random distribution with the non-zero precise $r = (r_1, \ldots, r_N)$ to $\theta_j$:

$$p(\theta_j | r) = \prod_{j=1}^{N} \mathcal{N}(\theta_{i,j} | 0, r_j)$$

(4)

Then with the transformation process in the literature \cite{13}, we can get the final probability distribution:

$$p(\theta_i | \lambda) = \int_0^{\infty} p(\theta_i | r) p(r | \lambda) dr = \frac{2^{\lambda/2}}{2^\lambda} \exp \left( -\sqrt{\lambda} \sum_j |\theta_{i,j}| \right)$$

(5)

Having defined the signal probability distribution model, we carry out the Bayesian inference by computing the posterior $p(\theta_i, r, \lambda, \beta | y_i)$. The type-II maximum likelihood approach is
used to perform the Bayesian inference in this paper. Firstly, we decompose the posterior \( p(\theta, r, \lambda, \beta | y) \) as

\[
p(\theta, r, \lambda, \beta | y) = p(\theta | r, \lambda, \beta, y) p(r, \beta, \lambda | y)
\]  

(6)

The \( p(\theta | r, \lambda, \beta, y) \) is the multivariate Gaussian distribution \( N(\theta | \mu, \Sigma) \) with the parameters like:

\[
\mu = \beta \Sigma_i \Phi_i^T y_i
\]

(7)

\[
\Sigma_i = (A + \beta \Phi_i^T \Phi_i)^{-1}
\]

(8)

where \( A = \text{diag}(1/r_1, \cdots, 1/r_N) \).

Now we utilize the posterior \( p(r, \lambda, \beta | y) \) to compute the hyperparameters. Exploiting

\[
p(r, \lambda, \beta | y) \propto p(y | r, \lambda, \beta) p(r) p(\lambda)
\]

we estimate the hyperparameters by computing the maximum of the joint distribution \( p(y | r, \lambda, \beta) p(r) p(\lambda) \), or equivalently its logarithm:

\[
L(r, \lambda) = \sum_{i=1}^{L} \log \left[ p(y_i | \theta, \beta) p(\theta | r) p(r | \lambda) p(\lambda) \right] d\theta
\]

(9)

with \( C_i = \beta^{-1} I + \Phi_i A^{-1} \Phi_i^T \). There are two approaches to maximize \( L(r, \lambda) \) with respect to \( r \) and \( \lambda \).

<1> Iterative Solution: We respectively differentiate (9) with respect to \( r \) and \( \lambda \), set the results to zero and have

\[
r_{\text{new}}^j = -L + \sqrt{L^2 + 4\lambda L \sum_{i=1}^{L} (\mu_{i,j}^2 + \Sigma_{i,j})}
\]

\[
\lambda_{\text{new}} = \frac{2(N-1)}{\sum_{j} r_j}
\]

(10)  (11)

where \( \Sigma_{i,j} \) is the \( j \)th diagonal component of the \( \Sigma_i \) of the \( i \)th signal and \( \mu_{i,j} \) is the \( j \)th component of \( \mu_i \) of the \( i \)th signal. We note \( r_{\text{new}} \) is the function of the \( \{\mu_i\}_{i=1 \cdots N} \) and \( \{\Sigma_i\}_{i=1 \cdots N} \). At the same time, \( \{\mu_i\}_{i=1 \cdots N} \) and \( \{\Sigma_i\}_{i=1 \cdots N} \) are the functions of \( r_{\text{new}} \). An iterative algorithm is performed among (7), (8) and (10). \( \lambda_{\text{new}} \) is get from \( r_{\text{new}} \) with (11). However, the iterative solution has a
limitation that it needs $O(N^3)$ operation. It makes that the approach is very slow when it is applied to a large-scale problem. So a fast algorithm is utilized in the following. The fast algorithm only needs $O(NK^2)$ operation, where $K$ is the sparsity of the original signal.

<2> Fast Algorithm: Considering that $\mathcal{L}(r, \lambda)$ dependents on the hyperparameter $r_j, j \in \{1, \cdots, N\}$, we can get

$$C_i = \beta^{-1} I + \sum_{k \neq j} r_k \Phi_{i,k} \Phi_{i,k}^T + r_j \Phi_{i,j} \Phi_{i,j}^T = C_{i,-j} + r_j \Phi_{i,j} \Phi_{i,j}^T$$

where $\Phi_{i,j}$ is the $j$th column of $\Phi_i$, and $C_{i,-j}$ is the contribution to $C_i$ without $\Phi_{i,j}$. Applying the above determination, the correspond parts in (9) can be written as

$$C_i^{-1} = C_{i,-j}^{-1} - C_{i,-j}^{-1} \Phi_{i,j} \Phi_{i,j}^T C_{i,-j}^{-1} \frac{1}{r_j + \Phi_{i,j} \Phi_{i,j}^T C_{i,-j}^{-1} \Phi_{i,j}}$$

(12)

$$|C_i| = |C_{i,-j}| \left| 1 + r_j \Phi_{i,j} \Phi_{i,j}^T C_{i,-j}^{-1} \Phi_{i,j} \right|.$$ 

(13)

Using the equation (12) and (13) and regarding $\mathcal{L}(r, \lambda)$ as the function of $r_j$, we can get

$$\mathcal{L}(r) = -\frac{1}{2} \sum_{i=1}^{L} \left( \log |C_{i,-j}| + r_y C_{i,-j} y_i + \frac{\lambda}{2} \sum_{k \neq j} r_k \right)$$

$$+ \frac{1}{2} \sum_{i=1}^{L} \left( \log \left( \frac{1}{1 + r_j s_{i,j}} \right) + q_{i,j}^2 r_j - \lambda r_j \right)$$

$$= \mathcal{L}(r_j) + l(r_j)$$

(14)

where $l(r_j)$ is $\frac{1}{2} \sum_{i=1}^{L} \left( \log \left( \frac{1}{1 + r_j s_{i,j}} \right) + q_{i,j}^2 r_j - \lambda r_j \right)$ and $r_j$ is $r$ without the $j$th component. $s_{i,j}$ and $q_{i,j}$ are defined as

$$s_{i,j} = \Phi_{i,j}^T C_{i,-j} \Phi_{i,j}, \quad q_{i,j} = \Phi_{i,j}^T C_{i,-j} C_{i,-j} \Phi_{i,j}$$

(15)

The maximum of $\mathcal{L}(r)$ with respect to $r_j$ is the same as the maximum of $l(r_j)$

Differentiating $l(r_j)$ with respect to $r_j$, we obtain

$$\frac{dl(r_j)}{dr_j} = \frac{1}{2} \sum_{i=1}^{L} \left[ -s_{i,j} + \frac{q_{i,j}^2}{1 + r_j s_{i,j}} \right] - \lambda \right]$$

(16)
In order to simplify the zero-finding procedure in (16), an approximation that \( r_j s_{i,j} \gg 1 \) is used in here. This approximation has been found to be valid numerically, e.g., typically \( s_{i,j} > 20 \times 1/r_j \). Therefore, setting (16) to be zero and getting the approximate result as

\[
\begin{align*}
    r_j & \approx -\left[ L + 2\lambda \sum_{i=1}^{L} \left( 1/s_{i,j} \right) \right] + \sqrt{\left[ L + 2\lambda \sum_{i=1}^{L} \left( 1/s_{i,j} \right) \right]^2 - 4L\lambda \sum_{i=1}^{L} \left( \lambda + s_{i,j} - q_{i,j}^2 \right)/s_{i,j}^2} \\
    & \quad \text{if } \sum_{i=1}^{L} \left( \lambda + s_{i,j} - q_{i,j}^2 \right)/s_{i,j}^2 < 0 \\
    & = 0 \quad \text{otherwise.}
\end{align*}
\]

(17)

The data from all the signals are used to update \( r_j \), and other processes of this fast algorithm, which is respectively performed on each signal, are same as the literature [13].

4. Simulation Result

The UWB channel used in the simulation is the LOS discrete multipath channel defined by IEEE, and meets the S-V channel model. The delay of multipath signals satisfies the Poisson process, and the attenuation of multipath components is the lognormal distribution. Suppose receiver receives \( L = 2 \) signals which have the same transmission channel model at the same time, namely, multipath components attenuation and arrival time of different signals meet the same probability distribution. The number of redundant dictionary atoms is assumed to be \( N = 2000 \). The project matrix used in Compressive Sensing reconstruction algorithm is different between users and both are random \( M \times N \) Gaussian matrix with i.i.d. draws of a zero-mean Gaussian distribution \( \mathcal{N}(0,1) \), and then the rows of matrix \( \Phi \) are normalized to unit. We add a zero-mean white Gaussian noise with standard deviation 0.005 to the measurement. The simulation is in the uplink scenario and made on MATLAB platform.

We select the second derivative of the Gaussian pulse as the signal pulse waveform which has the unit energy. The signal transmitted by different users has 2000 bits and the bits are generated randomly. Each bit is represented by one pulse. The PPM modulation scheme is adopted in the transmitter. The ideal RAKE receiver utilizes the compressive sensing algorithm to estimate channel and recover the signal.

Figure 2 compares the reconstruction performance of different compressive sensing algorithms, including basis pursuit algorithm (BP) [10] used in literature [4], Laplace prior based Bayesian Compressive Sensing (LBCS) [13], MT-BCS [9] and the proposed algorithm (D-LBCS), with the same UWB channel. Vertical axis is the successful probability of signal reconstruction (average 1000 trails), and horizontal axis is the number of measurement \( M \). The number of measurements changes from 200 to 500 with step 3. As can be seen from the Figure 2, under the premise that there is statistical correlation between the signals, the performance of the joint reconstruction is significantly better than the separate reconstruction. Because LBCS considers the prior probability distribution of signal, it outperforms BP. Between the joint reconstruction algorithms, the proposed reconstruction algorithm uses the Laplace prior, so it has the better recovery performance than MT-BCS and uses fewer measurements to achieve a satisfactory reconstruction effect.

Figure 3 shows that the signal recovery performance. The BER performance of different reconstruction algorithms is compared under different SNR. The number of measurement is fixed values \( M = 400 \). The yellow line is the traditional UWB communication system which
doesn’t use the compressive sensing algorithm. We can see that the UWB system based on CS has better BER performance than traditional UWB system. Among the UWB systems based on CS, the different performance of channel estimation results in the different signal recovery performance. Because of the good reconstruction result, the proposed algorithm shows the best BER performance.

![Figure 2. The successful probability of different CS algorithms](image1)

![Figure 3. BER performance under different SNR](image2)

5. Conclusion

Compressive Sensing theory can be used to solve the high sampling rate issue in the UWB communication system. In this paper, using the characteristic that the wireless channels of multiple user signals which are received by one receiver at the same time are statistically related in the multiuser UWB communication network system, we propose a Laplace prior based distributed Bayesian compressive sensing method. It jointly reconstructs the received signals and gets the channel model parameters. Comparing with other CS algorithms, the proposed method reduces the necessary sampling numbers for channel estimation. The experiment shows that the method improves the BER performance.
6. References


