Hairy Brush and Rice Paper Interactive Model with Chinese Ink Painting Style

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Abstract

It describes each component element of Chinese ink painting in detail. Firstly, this paper presents brush emulation based on physical calculation. The brush model comprises a spine skeleton wrapped up a smooth surface which can be deformed through constrained energy minimization. The model can bend the skeleton and change the shape of the tuft when there is a force applied on the brush. Then, it proposes the ink painting decomposition research technique, and proposes the simulation model system of the Chinese ink painting effect. This model had considered the writing brush model, as well as Xuan paper model of the water ink proliferation mode minutely. It proposed describes hairy brush behavior by the curving spring model on the base of the mechanical model simulation writing brush. This paper has considered the Xuan paper thickness influence to water ink proliferation, proposed the Xuan paper model to simulate by the fiber structure. In addition, it proposed the transfusion mechanical mode which considered the gluewater’s influence. So it had finished the ink proliferated interactive technology mode fusion physical and ink characteristic. It conformed to the Chinese ink painting unique style, and might carry on the effect according to the parameter the control, which caused the ink painting more vivid.

Keywords: Calligraphy Painting, Brush Model, Computer Simulation, Ink Diffusion

1. Introduction

For the Chinese people, calligraphy is an age-old traditions art, Common in the art curriculum to use is called the "four treasures" of the writing tools: hairy brush, ink stone, paper and ink, it serves to show the importance of calligraphy. But most people use brush exercises word experience also only in middle and primary school age, the reason is nothing else but a busy life. For non-technical terms for anyone use brush exercises word, the trouble lies not in written form, and practicing in the length of days, but in all kinds of material preparation, and writing after work. If we turn the computer would easily practiced writing, do not need as traditional Chinese calligraphy that preceded must be ready to laying cloth and four treasures, or even grinding ink, afterwards also cleaning. Using the computer simulation of words, just ready input device, open the computer can. So we want to build simulation model of the brush, hoping to allow users to easily use.

The Chinese calligraphy is one of the quintessence in Chinese art. Calligraphers use the Chinese brush stroke to write glamorous characters. As the brush moves, we can see the affections of the calligrapher/artist and his unique artistry. In other words, calligraphers express their attitude of mind and values by writing characters. In Chinese calligraphy, students who want to write beautiful characters like that written by famous calligraphers should imitate the copybooks first. After practicing repeatedly, they could find out the extensive knowledge and profound scholarship of the Chinese calligraphy, and create their own styles. The purpose of this thesis is to synthesize the specific Chinese calligraphic style of one of the famous calligraphers mentioned before. Our system also provides a convenient environment for users to imitate copybooks of calligraphy by computer techniques.

There are lots of researches that aimed to generate realistic calligraphic character images. Most of them focused on the physical properties of the Chinese brush stroke. Users who want to write a calligraphic character using those systems need to have enough basic knowledge of calligraphy. And

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some special equipment (including the real Chinese brush stroke connected to the computers) is also need. Thus, it is not convenient for most users.

Chinese artists and poets use hairy brush to write calligraphy and draw water ink paintings. With the development of computer science, many researchers tried computerizing the hairy brush for convenient, practical, and realistic usage. In this paper, we present a model of emulating the hairy brush and synthesize corresponding calligraphy. It is highly complicate to model a real hairy brush. In Chinese painting and calligraphy, a painter or calligrapher only uses several simple strokes to present his emotions, feeling, and the artistic spirit. Modeling a 3-D hairy brush in order to satisfy those moves which the painter want is very difficult. We only try to model a brush with physical properties like a real brush as much as we can, based on knowledge from the paper of an efficient brush model construction [1], particularly the bending property of the brush when we apply a force on the brush.

2. Related works

There are a lot of researches that aim to the topic of brush model. Brushing commonly refers to the drawing of curves with various line widths in bit-mapped graphical systems. In 1989, Porch and Fellner proposed the Circle-Brush Algorithm [2]. This approach produce constant line width by circles of suitable diameter, independent of the curve’s slope. At the same time, Strassmann’s hairy brushes model [3] was presented. It provided a description of physical properties of brush materials in order to generate hairy brush images of sumi-e paintings – one kind of traditional Japanese art. And the concept of a collection of bristles was proposed for the first time.

After that, Horace and Helena [4] proposed the first methodology for generating hairy-brush writings. A parameterized model is built to specify the varying brush orientation and brush tip pressure, the brush writing hair properties and the variation of ink deposition along a stroke trajectory. From this model, people can simulate the physical process of brush stroke creation and synthesize most of the aesthetic features of calligraphic writings. This method is suitable for Chinese calligraphy.

Then, Nelson et al. [5] presented a 3D brush model. The main feature of this model is its ability to mimic brush flattening and bristle spreading due to brush bending and lateral friction exerted by the paper surface during the painting process. Since the visual feedback is significant in their system, some special equipment is required such as a real brush and haptic device.

Most researches about Chinese calligraphy are devoted to image process of the calligraphic documents. Yang et al. [6] proposed a method to vectorize the digital images of the Chinese characters automatically. The vectorization results can be transformed into a true-type font for general applications. They also prevent the zigzag phenomena when enlarging the characters. In this way, the treasures of the arts of Chinese culture can be preserved.

A different kind of application on calligraphy and image processing was suggested by Wei et al [7]. They proposed a method to generate scratched look calligraphy characters by mathematical morphology. By this method, people can decide on number of times of the thinning computation and structuring elements, and can also know whether the sizes of generated calligraphy characters are the same as the original one in theory.

A combination of brush and image process was presented by Mi et al [8]. They proposed a virtual brush model based on droplet operation and its application on retrieving character outlines and character modeling in Chinese calligraphy style. The droplet model helps to compute stroke area with well-defined geometry information and leads to the feasibility of retrieving the outlines of characters with well-defined geometry representation.

There are many brush models which were designed previously. One of the earliest models was developed by Strassmann in which he modeled a brush as a one-dimensional array of bristles swept over a trajectory defined by a cubic spline curve [9]. This model could account for varying color, width, and wetness. It is effective approach but not easy to use for non-computer specialists. Wong and Ip defined a complex set of interrelated parameters to vary the density, opacity and shade of a footprint of the current brush draw mark which take into account the behavior of a three-dimensional round calligraphy brush [10]. This represented a substantial improvement over Strassmann’s in term of usability. Using the theory of elasticity, Lee [11] modeled a brush as a collection of rods with homogeneous elasticity along the entire length. This model suffers from unnatural bending because it
assumes homogeneous elasticity. Saito et al. [12] used a Bezier spine curve and a set of discs centered along the curve to model a brush. However, this model doesn’t consider the brush flattening and spreading and thus fails to generate a realistic footprint. In the DAB project [13], a subdivision surface is wrapped around a spring-mass particle system skeleton to represent the brush geometry. Using an approximated implicit integration method, they were able to produce a real-time system for doing acrylic-like painting. To generate the subdivision surface to model the brush head, either interpolation or some approximating scheme is used. An interpolated brush head, however, often cannot deform smoothly because of frequent occurrence of high curvature in the brush head surface, while approximation has the problem of properly placing control points to yield the desired surface. In the work by Xu et al. [14], general sweeping is employed to establish the solid geometry model of each hair cluster. The problem however is that general sweeping is a time-consuming operation. At different stages of painting, much computation is needed to apply general sweeping operations to update the model. Also, the solid model requires a fair amount of memory for its internal representation, and after the brush is split many times, the demand for memory could become a bottleneck. In the work by Chu and Tai [5], a single hair bundle is modeled by a geometry model that is mathematically equivalent to that by Xu et al.’s. Unlike the latter which simulates the spreading and splitting of the brush tip by a geometry approach, Chu and Tai use an alpha map to implement cluster modeling for the split brush. In the recent work of Xu et al. [15], a hierarchical representation is applied on geometry model, that leads to substantial savings in every step of the painting process; online brush motion simulation assisted by offline calibration that guarantees an accurate and stable simulation of the brush’s dynamic behavior. They create a new pigment model based on a diffusion process of random molecules which consider delicate and complex pigment behavior at dipping time as well as during painting. However, their system needs the assistance from offline calibration, thus choosing the appropriate samples for the brush motion calibration database which enumerate all the possible motions of painting brush will be a problem.

3. Problem description

3.1. Brush mechanical model

Construction of a brush model includes the geometric model of the building and the construction of mechanical models in two parts. In order to meet the needs of real-time interaction in the simulation at the same time to meet two conditions: first is the computing time required can not be too long in order to immediately respond to a user’s behavior, in order to meet the real-time, this paper uses the hardware-accelerated system; second one is better than stability, the user may make a variety of actions, and brush the model in this case still must be able to maintain stable operation.

Written skeleton brush can be used to simulate particle system. Will be written as a number of particles in the middle of spring to connect, and each particle of that part of the representative of the quality of the mid-point. Behavior of particle systems can be used Newton's laws of motion to describe. Assume that $n$ particle, particle coordinates $x$, the velocity of particle $v$, forces on particles for $F$ ($x$, $v$, $F$ are 3D vector), it is assumed that the weight of each particle equal to $M$. Can be expressed as two-order differential equations:

$$
\begin{align*}
\ddot{x} &= v \\
\dot{v} &= F / M
\end{align*}
$$

In order to approximate the solution of the differential equations, using the following method:

a) The linear part of the edge points so hidden (Implicit Euler Integration);
b) Non-linear part of the force to do the approaching post-correction;
c) Adding restrictions to prevent excessive flexion and extension springs.

By using this method of seeking out solutions, although not the most accurate, but because it separated from the linear part of the direct count, followed by further amendments to the rear of the action, so the stability of the faster and higher, just to meet the real-time requirements.
The so-called implicit integration method with the explicit integration method (Explicit Euler Integration) the difference is: in the calculation of the speed of a unit time, the dominant points is used to calculate the force now, and the hidden points are used the next unit time to calculate the force.

Explicit integration:

$$V_{t+1} = V_t + F_t \times \frac{dt}{M}$$  \hspace{1cm} (2)

$$X_{t+1} = X_t + V_{t+1} \times dt$$  \hspace{1cm} (3)

Implicit integration:

$$V_{t+1} = V_t + F_{t+1} \times \frac{dt}{M}$$  \hspace{1cm} (4)

$$X_{t+1} = X_t + V_{t+1} \times dt$$  \hspace{1cm} (5)

Although the dominant points is easier to implement, but its lack of stability than that. Therefore the use of implicit integration method, it is necessary to implement this approach, first of all to the next time do not know the location of the point situation, calculate the next point in time, and the use of Taylor's estimate of the value of a start:

$$F_{t+1} = F_t + \frac{\partial F}{\partial X} \times (X_{t+1} - X_t)$$  \hspace{1cm} (6)

Will be (6) into (4), let \( H = \frac{\partial F}{\partial X} \),

$$V_{t+1} = V_t + [F_t + H(X_{t+1} - X_t)] \times \frac{dt}{M}$$  \hspace{1cm} (7)

Then (5) into (7), acquireability:

$$V_{t+1} = V_t + (F_t + H \times V_{t+1} \times dt) \times \frac{dt}{M}$$

$$\Rightarrow V_{t+1} \times (I - H \times \frac{dt^2}{M}) = V_t + F_t \times \frac{dt}{M}$$

$$\Rightarrow V_{t+1} = (I - H \times \frac{dt^2}{M})^{-1} \times V_t + (I - H \times \frac{dt^2}{M})^{-1} \times F_t \times \frac{dt}{M}$$

Finally gets

$$\Delta V_{t+1} = V_{t+1} - V_t = (I - H \times \frac{dt^2}{M})^{-1} \times (F_t + H \times V_t \times dt) \times \frac{dt}{M}$$  \hspace{1cm} (8)

Finally, the mechanical equation becomes:

$$\begin{pmatrix} \Delta X_{t+1} \\ \Delta V_{t+1} \end{pmatrix} = \begin{pmatrix} (\Delta V_{t+1} + \Delta V_t) \times dt \\ (I - H \times \frac{dt^2}{M})^{-1} \times (F_t + H \times V_t \times dt) \times \frac{dt}{M} \end{pmatrix}$$  \hspace{1cm} (9)

Because only part of the hidden points dominant, so \((I - H \times \frac{dt^2}{M})^{-1}\) is a constant, you can count in advance, and save each time to count the anti-matrix.

Aristotle is not due to mechanical inertia of the existence, meaning that determines the strength of the objects movement, once the forces disappear, objects will stop. Experiments show that Aristotle than Newton's mechanics better adapted to simulate the mechanical behavior of brush written, but also
high speed and stability advantages. In Aristotelian mechanics, the particle system into a physical formula:

$$\& x = \frac{F}{M}$$ (10)

Based on Eq. (4), (6) similar hidden derivation process of integration, let $H = \frac{\partial F}{\partial X}$, so

$$X_{t+1} = X_t + (F_t + H \times V_{t+1} \times dt) \frac{dt}{M}$$

$$\Rightarrow X_{t+1} \times (I - H \times \frac{dt}{M}) = X_t + F_t \times \frac{dt}{M} - H \times X_t \times \frac{dt}{M}$$

$$\Rightarrow X_{t+1} = (I - H \times \frac{dt}{M})^{-1} \times X_t + (I - H \times \frac{dt}{M})^{-1} \times (I - H \times \frac{dt}{M})^{-1} \times X_t \times \frac{dt}{M}$$

$$\Rightarrow X_{t+1} = (I - H \times \frac{dt}{M})^{-1} \times X_t + (I - H \times \frac{dt}{M})^{-1} \times F_t \times \frac{dt}{M}$$

$$\Delta X_{t+1} = X_{t+1} - X_t = (I - H \times \frac{dt}{M})^{-1} \times F_t \times \frac{dt}{M}$$ (11)

Finally,

$$\Delta X_{t+1} = (I - H \times \frac{dt}{M})^{-1} \times F_t \times \frac{dt}{M}$$ (12)

Through careful observation physical behavior of pen hair, we will be found in the fact that it compared the characteristics of spring as bending rather than stretching the spring. Brush is made of animal hair, while having the flexibility, but it is still hard, and will not change with the force and length changes, but change the shape by extrusion, there will be trends in restitution. Therefore, the spring bending simulation is used for hair brush pen.

Spring bending formula:

$$F = k(\theta - \theta_0)$$ (13)

$$\Delta \theta_{t+1} = (I - H \times \frac{dt}{M})^{-1} \times F_t \times \frac{dt}{M}$$ (14)

3.2. Brush geometry characteristic behavior

The hairy brushes, used in Chinese painting and calligraphy, are made from hair of animals, such as horse, deer, and rabbit. According to the kinds of hair, they are generally classified into three main types: hard, soft and combination, each of which has different stiffness and absorbent. The bristles have different length; shorter ones are inside and longer ones are outside. The brush forms a single tuft and run into a fine tip when it is moisten. We will emulate the brush in moisten state, not in the dry state. A brush is elastic. It bends when some external force applied to it and restore to its original shape when the force is released. Emulating this deformation in real time is a tough problem. To make problem simpler, we model the brush with a simple skeleton and compute the deformation of this skeleton and apply it to the whole brush [16], as shown in Figure 1. Our emulation includes two parts: brush geometry and brush dynamic.
3.2.1. Brush geometry

The better brush geometry presents, the better brush dynamic works. To model a flexible model, we divide brush geometry into two layers, brush skeleton and brush surface [13].

The brush skeleton is divided into six segments, which have shorter length forward the tip. Since the brush root is stiffer than the tip, we decrease the length of segments forward to the tip to make the brush tip bend more. These segments are connected to each other with spine nodes, Figure 2. The first spine node is attached to the brush handle. With 6 segments, we have 7 spine nodes, \( N_0, N_1, \ldots, N_6 \). We denote the positions of 7 spine nodes are \( O_0, O_1, \ldots, O_6 \) and the bending angle, the angle between the current segment and its previous segment, at spine nodes are \( \theta_0, \theta_1, \ldots, \theta_6 \), respectively.

Beside the spine nodes, we also define lateral nodes to emulate the brush better. Each spine node has two lateral nodes. These two nodes will run along the line passing through the spine node on the joint-bisecting plane. This plane is the crossing plane pass through spine node and bisecting the angle between two adjacent segments and perpendicular with the plane spanned by the segment [5]. The different movement of two lateral nodes of two side of spine node will make the brush deformation more flexible. It shows as Figure 3.

The brush surface is a sweep surface, shown in Figure 4, created by running an ellipse cross section, along the brush skeleton. When brush is in original state, i.e. there is no force applied to the brush, the cross section is a circle. We predefine the radii of these circles. The cross section is actually two half ellipses, which have the same minor radius but have different major radii. To create the surface, we form the brush skeleton by cubic b-spline method and let the cross section run along the skeleton. The radius of the cross section at the spine node \( i \) are calculated as follows: with the center of two half ellipses is the spine node \( i \), let \( a \) and \( b \) are the distance between the spine node to two lateral nodes, the major radii of two half ellipses are \((a+r_i)\) and \((b+r_i)\), where \( r_i \) are the radii of lateral nodes. According to the unchangeament of circle/ellipse are, the minor radius is:
3.2.2. Brush dynamic

Our brush model has to be bended when it touch the paper with a given applied force and return to normal state when the force release. This includes conservative and dissipative forces, and can be referred as incremental potential energy. To model the brush dynamic, we use energy minimization method to define the deformed state of the brush tuft at a certain time frame. With a certain applied force, we formulate an energy minimization function with Newtonian physics, then solving this constraints optimization problem and updating the state of brush.

First, we will describe the algorithm for brush dynamic problem in Figure 5. Then we will present energy minimization problem. Let $O_i$ $(i=0, 1, \ldots, 6)$ is the position of spine nodes, $\theta_i$ is the angle between the $i$-th segment and the previous segment. We can get the new position of the root spine node, $p$, from tablet device. The algorithm is as follows [5]:

a) Calculate the distance between the new position and old position of the root spine node, let $O_0 = p$, where $p$ is the new position.

b) Determine if there is any spine nodes penetrate the paper. If yes, set minimization constraints of these spine nodes to be above the paper.

c) Solving the energy minimization problem to get $\theta_i$ and update the new position of spine nodes.

The energy function is used to demonstrate the deformation of the brush when there is a force applied on it. Applied the minimum principle of incremental potential energy [17], we can predict the shape of deformed brush. The incremental potential energy terms here account for strain energy and friction force. At each time frame, the program update the brush state through the deformation of brush skeleton which satisfying all constraints.

The brush dynamics is a series of static constrained minimization problems. The objective function equation (15), i.e. the function needed to be minimized, is the dynamic system’s current incremental potential energy, and the constraint is that the brush geometry must not penetrate the paper. The set of variables in the minimization formula comprises all the joint angles of spine nodes. These are the factors related to brush dynamic: friction energy and strain energy. The strain energy accounts for the potential energy stored in the brush due to deformation from its unstrained state. The friction energy includes internal frictional energy and external frictional energy [17]. The former refers to the work...
done against the internal friction between the water and bristle molecules during tuft deformation. The later refers to the work done against the frictional force between the brush and the paper surface.

\[ E = E_{\text{deform}} + E_{\text{friction}} \]  

(16)

3.3. Brush model constraints minimization function

The nonlinear optimization problem has general formula:

\[
\begin{align*}
\min / \max \quad f(x) & \quad x \in \mathbb{R}^2 \\
g_i(x) &= 0 \quad i \in I \\
h_i(x) &= 0 \quad i \in E \\
I_i &= x = E_i \quad i = 1, 2, \ldots, n
\end{align*}
\]  

(17)

Where \( I \) and \( E \) are finite non-negative integers subsets, and function \( f(x) \) is object function which need to be optimized, \( g_i(x) \) and \( h_i(x) \) are constraints functions, \( I_i \) and \( E_i \) are lower bounds and upper bounds of variable \( x \).

To solve the optimization problem, we apply KNITRO package, which implements three state-of-the-art, interior-point and active-set methods [18]. Each algorithm possesses strong convergence properties. Those three algorithms are Interior/Direct algorithm, Interior/CG algorithm, and Active Set algorithm and described as followed:

a) Interior/Direct algorithm: Interior-point methods (also known as barrier method) replace the nonlinear programming problem by a series of barrier sub-problems controlled by a barrier parameter \( \mu \). Trust regions and a merit function are used to promote convergence. Interior-point method performs one or more minimization steps on each barrier sub-problems, then decrease barrier parameter and repeat the process until the original problem equation (16) has been solved to the desired accuracy. The Interior/Direct method computes new iterates by solving the primal-dual KKT matrix using direct linear algebra.

b) Interior/CG algorithm: This method is similar to the Interior/Direct algorithm, except the primal-dual KKT system is solved using a projected conjugate gradient iteration. This approach differs from most interior-point methods proposed in the literature. A projection matrix is factorized and conjugate gradient applied to approximately minimize a quadratic model of barrier problem. The use of conjugate gradient on large-scale problems allow the algorithm utilize exact second derivatives without forming the Hessian matrix.

c) Active Set algorithm: Active set methods solve a sequence of sub-problems based on a quadratic model of the original problem. In contrast with interior-point methods, the algorithm seeks active inequalities and follows a more exterior path to the solution. It implements a sequential linear quadratic programming (SLQP) algorithm, similar nature to a sequential quadratic programming method but using linear programming sub-problems to estimate the active set. This method may be preferable to interior-point algorithms when a good initial point can be provided; for example, when solving a sequence of related problems.

Applying these methods often takes from 18 to 27 iterations to solve our energy minimization problem. The Figure 6 below shows one of results of the energy calculation.
4. Interactive Xuan paper model

Simulating the behavior of Chinese ink is challenging work because ink moves in a complex manner. This paper proposes a new method for simulating the diffusion of ink in paper. The method is based on a physical mechanism and an observational model of the interaction among real drawing materials used in Chinese ink painting and the variations in the diffusion of ink in the real world. The goal is to capture the core physical properties and behaviors to produce a high-quality ink diffusion model that a painter can use to generate a Chinese ink painting, including brush strokes, in various styles.

The proposed method has the following advantages. First, it simulates the physical behavior of ink diffusion, and can thus generate strokes that exhibit a feathery effect; second, it can blend two strokes with different thickness. Using this method to render a Chinese ink painting can generate highly realistic blending effect.

4.1. Discrete paper model

Several kinds of paper are used in Chinese ink painting. Basically, papers have one of two types of fiber mesh. The first kind is a regular fiber mesh, such as in silk paper, whose fibers are uniformly aligned as woven. The second kind is an irregularly distributed fiber mesh, such as in Xuan paper that consists of a mesh of randomly positioned fibers.

Constructing a mesh like Xuan paper requires an appropriate data format in which to represent a mesh structure. Traditionally, a network format is used to represent paper with a random fiber network [19]. The continuous interaction between water and fiber is discretely simulated by computers; a two-dimensional array, whose entries specify the attributes of the structure of the mesh, is used. The entire mesh in the paper is separated out into many layers, each of which is divided into \([X\times Y]\) cells called papels (paper element) [20]. A papel is a basic unit of paper structure and corresponds to a pixel.

Capillarity is evident in paper modeled as interlaced fibers. The ink seeps into the paper and is then pulled away from the area of application by capillary attraction; it then travels through the fibers. Some of the diffused ink is deposited in the holes or spaces between the fiber through which it passes; the remaining ink continuously flows along the fibers until it is completely absorbed.

Let Absorbency(\(p\)) of papel \(p\) be defined as follows. When the moving ink passes through \(p\) with \(N\) fibers, the amount of water deposited in \(p\) is \(Q\). The relationship between \(Q\) and \(N\) can be expressed as

\[
\text{Absorbency}(p) \propto N \propto Q.
\]

Based on this relationship, several models of paper can be defined with various absorbency, by fibers with various densities. An equation for the absorbency of each papel is,

\[
\text{Absorbency}(p) = B_{\text{base}} + \text{Var} \times \text{rand}() \tag{18}
\]

The texture of the paper also determines the directions in which the water flows. When water undergoes capillary action and flows in the holes among the fibers, fibers in the trajectory of the flowing water become saturated. Variously aligned of fibers promote different trajectories of the flowing water. Two kinds of paper exist—one with regular and the other with an irregular fiber mesh, such as silk paper and Xuan paper, respectively.

Papers with differently distributed fibers have different textures. When the water particles in a papel \(C_0\) flow out, \(C_0\) is the center of a \(3\times3\) texture mask, 
\(M_{\text{direct}}\) with a central element \(m_0\) at \(c_0\). The eight
elements at the periphery of $M_{\text{direct}}$, $m_k$ ($k=1, 2, \ldots, 8$), are assigned weights to represent the alignment of the fibers.

4.2. Ink diffusion process

The diffusion of ink in paper is complex. It can be regarded as a continuous and time-dependent. The evaporation of water and the absorption of any ink left on the surface of the paper are also considered.

Ink diffusion is caused by the capillary action of water between fibers and the gradient of the quantity of water in paper cells. Given two neighboring papels saturated with water, as a rough estimate, only the strength of the capillary forces in these two papels influences the direction of flow of the water. In contrast, only one papel is saturated with water and the other is absolutely dry. The water gradient between these two papels is maximal, resulting in an obvious propagation of water from the papel with much to the other with little.

Besides, three issues were addressed in simulating ink diffusion brush strokes, initial area and propagation. The area on the surface of the paper touched by the brush is approximately circular, because the profile in the horizontal direction of the brush used in Chinese Ink Painting is round. The stroke is a sequence of circular segments shown in Figure 7. Ink in old segments starts to diffuse earlier than in new segments, and ink within old segments may even dry up. Stroke segmentation makes the simulation of the painting processes more realistic. The skeleton of a brush stroke area of the application of ink is just one line, and can be simply used to describe a brush stroke as a trajectory of the center of a circle.

The application of the brush is an essential element of landscape painting techniques. Brushwork is very important in Chinese ink painting. A brush consists of a bundle of bristles. Where the brush contacts the rice paper, the footprints of the bristles form a contact region. While painting, these bristles deposit ink on the circular region in contact with the Xuan paper, as shown in Figure 8.
The contact region is a 2D ellipse. Consider in Figure 8, an ellipse $C$, with center $O$, as the contact region between the brush and Xuan paper. $A$ and $B$ are the short axis and long axis of $C$, respectively. A set of bristles is distributed inside the circle. We denote each bristle as $b_i$, and represent it in polar coordinates with respect to the center of the ellipse $C$.

$$b_i = (d_i, \theta_i)$$  \hspace{1cm} (19)

Where $i$ is the bristle index, $d_i$ is the distance from $o$ to $b_i$, $\theta_i$ is the angle between x-axis and $\overline{O b_i}$. Herein, the central angle of $C$ is split equally to obtain the base angle, $\theta_{\text{base}}$, such that $\theta_i = i \cdot \theta_{\text{base}}$. For each $\theta_i$, we locate the number of bristles on the radius. Two parameters allow the number of bristles to be controlled: one is the base angle $\theta_{\text{base}}$, and the other is the number of points located on the radius. A smaller base angle and more located points both result in more bristles. Parameters may be adjusted to obtain different brush sizes and resolutions.

5. Implementation and evaluation

We have rendered the brush model using OpenGL bound with Visual C++ in general purpose PC platform with an Intel dual core 2.4GHz CPU and 1GB RAM.

The system architecture includes three phases: input data, emulation of the brush, expert system, stroke and background environment rendering as shown in figure 9. The system provides a convenient user interface and service for intelligent decisions, has full knowledge base and provide.

![System Implementation Process](image)

**Figure 9.** The system implementation process

This phase is actually the emulation of the behavior of the tuft represented by a bristle spine. Based on the force and solving constrained energy minimization functions. Brush and Xuan paper’s interactive model shows in Figure 10.

![Interactive Structure Diagram](image)

**Figure 10.** Interactive structure diagram

With coordinates of spine nodes at a certain time frame, we can use OpenGL to render the brush model and background environment. In the first module, we input the stroke trajectories which can be done with a digitizing pen or a mouse to start the calligraphy system.

To obtain footprint, we let a part of the brush surface intersect the paper plane, and calculate the orthogonal projection of the penetration. We only need to calculate the penetration of ellipse cross sections which intersect the paper line. After getting all cross points, we will have the footprint of the
brush. In the second module, the relationships among all strokes are determined. Finally, when each stroke is allocated with a specific stroke type, we can construct the whole character and show the result in Figure 11.

Figure 11. Brush model whole stroke simulation result

6. Conclusion and future works

We have presented a model for 3D hairy brush emulation. This interactive model can real time deforms the shape of brush tuft when there is a force applied on, based on constrained energy minimization function. However, the main feature of calligraphy system is the integration of computer graphics and knowledge engineering. With computer graphics, the graphical behavior of calligraphy can be presented in the virtual world. With knowledge engineering, the aesthetic calligraphic characters can be generated intelligently. We combine these two different fields in computer science to achieve better results.

7. References