Study on Initial Schedule Optimization Model of Intercity Passenger Trains based on ACO Algorithm
Dingjun Chen, Miaomiao Lv, Shaoquan Ni
School of Transportation and Logistics, Southwest Jiaotong University, Chengdu, China, 610031
E-mail: chen-dingjun@163.com

Abstract

The initial schedule of passenger trains is the base of train working diagram. The problem of initial schedule of passenger trains is translated to the n-level decision-making problem in the paper. Then we proposed a heuristic ant colony optimization algorithm to resolve the problem and accelerated the speed of the ants search the optimal paths based on the neighborhood of the point. Furthermore, some constraint process strategy are introduced to improve the algorithm. Finally, the example is proposed to prove the effectiveness of the model and algorithm.

Keywords: Initial Schedule, Fuzzy Sets, Passenger Trains, ACO Algorithm

1. Introduction

The initial schedule of passenger trains is the skeleton of passenger train working diagram. Rational determination of the initial schedule of passenger trains, namely rational determination of passenger train’s departure time and arrival time, is the important measures for attracting passengers, and improving service quality of passenger trains, more, how to determine it rationally is deserve attention in compiling train running schedule.

There have been many scholars have discussed how to run scheme of passenger trains from different perspectives. Fu Zhuo[1] presented a new optimal algorithm for the problem with the combination of qualitative analysis. Sun Yan[2] gave a mathematics model and induced a three sub –programs from the main program. Ma Jiangjun[3] bases on mode of organization of the middle-speed train running on the high-speed railway network and studies the calculation methods of the scopes of the originating time and the terminating time of the changing-line middle-speed train which runs on the existing railway line. Liu Aijiang[4] proposed a new paired train model based on genetic algorithm on single-track lines. Chen Tuansheng[5]taking the maximum degree of passenger travel convenience as the objective function and considering some constraint conditions such as the carrying capacity of arrival and departure tracks and passenger trains must departure and arrive in rational time intervals, an objective programming model for optimizing the departure and arrival time intervals of passenger trains is constructed. Chen Lingling[6] used the congruence theory to investigate the time interrelation for two passenger trains from up and down direction in a big station to shorten the change and ride time of passengers in a transfer station. Shi Feng[7] taking the minimum degree of passenger’s travel costs as the objective function, a bi-level programming model was designed for optimizing the departure time distribution of passenger trains was constructed and an optimal algorithm based on the simulated annealing algorithm. Ni Shaoquan[12] established an objective programming model for optimizing the initial schedule of passenger trains, and puts forward a method for the model which is basing on improved Genetic Algorithm.

Literature mentioned above have carried on the beneficial exploration to the initial schedule of passenger trains, but existing mathematics models are designed mainly considering how to facilitate passenger, while passenger convenience and passenger station’s capacity are not considered comprehensively. More, reasonable travel time for passenger changes is a time range, existing deterministic mathematics models can’t measure reasonably. In view of this, based on fuzzy set theory, comprehensively considering some constraint conditions such as passenger station’s capacity and train-set joining time, etc, a mathematics model for optimizing the initial
2. Mathematics model for optimizing the initial schedule of intercity passenger trains based on fuzzy sets

2.1. Basic concepts and conclusions

Definition 1: Let \( U \) be a universe, \( A \) is a subset of. For any \( x \in U \), function \( \mu_A : U \rightarrow [0, 1] \), then \( A \) is called fuzzy set. \( \mu_A(x) \) is called membership function, which expresses the degree of \( x \) belong to \( A \).

Definition 2: Fuzzy expectation of passenger trains’ time: passenger’s expectation departure time range is \([F^1, F^4]\), the level of service satisfaction is lower if time is outside the range, specially, if trains’ departure time is in \([F^2, F^3]\), the degree of passenger satisfaction is highest. Suppose \( x \) is the time for passenger departure, membership function of satisfaction of passenger’ fuzzy expectation time is defined as follows:

\[
\mu(x) = \begin{cases} 
\frac{x - F^1}{F^2 - F^1}, & \text{if } F^1 \leq x \leq F^2 \\
1, & \text{if } F^2 \leq x \leq F^3 \\
\frac{x - F^4}{F^3 - F^4}, & \text{if } F^3 \leq x \leq F^4 \\
0, & \text{else}
\end{cases}
\]

2.2. Mathematics model

Max

\[
\sum_{i=1}^{s} \mu(t_{i}^{'}) + \sum_{r=1}^{2x} \mu(t_{r}^{'})
\]

s.t.

\[
\left| t_{i,j}^{' -} - t_{i,j}^{' +} \right| \geq I_{\text{departure}} \quad (1)
\]

\[
\left| t_{i,j}^{' -} - t_{i,j}^{' +} \right| \geq I_{\text{arrival}} \quad (2)
\]

\[
t_{i,j}^{' -} - t_{j,i,j-1}^{' -} \geq \xi_{j-1,j} \quad (3)
\]

\[
t_{i,j}^{' +} - t_{j,i}^{' +} \geq \xi_{i,j} \quad (4)
\]

\[
t_{i,j}^{' +} - t_{i,j}^{'} \geq T_{ij}^c \quad (5)
\]
Study on Initial Schedule Optimization Model of Intercity Passenger Trains based on ACO Algorithm
Dingjun Chen, Miaomiao Lv, Shaoquan Ni

\[ t_{i_w}^e = t_{i_1}^e + \sum_{k=1}^{n-1} \sum_{j=1}^{n-1} T_{ij} \] (6)

\[ t_{i_j}^e = t_{i_1}^e + \sum_{k=2}^{n} \sum_{j=1}^{n} T_{ij} \] (7)

\[ \sum_{j=1}^{n} \left[ w(t_{i_j}^e, t) - w(t_{i_j}^e, t) \right] \leq Ns_j \] (8)

\[ \sum_{j=1}^{n} \min \left\{ t_{i_j}^e - t_{i_j}^e, w_{i_j}^e - t_{i_j}^e \right\} \geq t_{i_j \cap \alpha \cap \gamma}^e, i = 1 + n, ..., 2n \] +

\[ \sum_{j=1}^{n} \min \left\{ t_{i_j}^e - t_{i_j}^e, w_{i_j}^e - t_{i_j}^e \right\} \geq t_{i_j \cap \alpha \cap \gamma}^e, i = 1, ..., n \] \leq \beta T_{i_j \cap \alpha \cap \gamma} (9)

Where the objective function represents maximize the Satisfaction of passengers expectation departure time constraint. Constraint (1) and (2) represent minimum secure time interval. Constraint (3) and (2) represent train running time between departure station and arrival station. Constraint (4) represents stopping time at station of passenger trains. Constraint (6) and (7) represent train running time calculation relationship equation between the departure time and the arrival time of passenger trains. Constraint (8) represents the number of arrival and departure tracks occupied by trains is not more than the number of station tracks. Constraint (9) represents train-set joining time.

3. ACO algorithm for initial schedule of intercity passenger trains based on fuzzy sets

3.1. Solution structure

From the feasible solution space (Fig.1), for any \( x_i \), there is \( l \) points to choose. All the variables get the value that can get a solution. Thus the \( (x_1, x_2, \cdots, x_n) \) is a solution while \( l = b - a + 1 \), \( a, b \) are integer and the unit is minute, \( [a, b] \) express the time range of intercity passenger train in the problem. If \( x_i \) choose the \( h_i \) point, then \( (x_1, x_2, \cdots, x_n) = (a + h_i - 1, a + h_i - 1, \cdots, a + h_i - 1) \). This means the initial time of trains \( i \) is \( a + h_i - 1 \).

*Fig 1. Feasible solution space*
3.2. Transition probability

The problem translate to the n-level decision-making problem and every level has \( l \) points. The probability of \( j \)th level choose the \( i \)th point is defined as:

\[
p_{ij} = \frac{\tau_{ij}}{\sum_{i} \tau_{ij}}
\]

While \( \tau_{ij} \) represents the pheromone mount which can understand as the attractive of \( j \)th level choose the \( i \)th point. And the updating rule is showed in follow:

\[
\tau_{ij}^{new} = \rho \cdot \tau_{ij}^{old} + \frac{f}{Q}
\]

While \( \rho \) represents the evaporate speed of pheromone (\( 0 < \rho < 1 \)) general choosing 0.5 to 0.9. \( Q \) is a constant, representing the total pheromone amount of one ant. \( f \) is the value of objective function.

3.3. Constraint processing method

1. Neighborhood of point: in the problem, we define the neighborhood of point \( x \) as the interval \((x-I, x+I)\), note \( O(x) \). We can use the concept that ensure the constraint1 and constraint2. Once the points are selected in previous level when the ant searching the path, then the ant in the next level can’t choose the neighborhood of these points. Also the method can increase to get the optimal solution.

2. If some trains have the strict special requirements with the necessary in the time range \([F^2, F^3] \)

we can modify the Membership function as

\[
\mu(x) = \begin{cases} 
1, & \text{if } F^2 \leq x \leq F^3 \\
0, & \text{otherwise}
\end{cases}
\]

3.4. Heuristics ACO algorithm

**Step1** initialize the parameters: the related parameter is as follows:

1. the cycle numbers (\( k = 0 \) );
2. Evaporation value of pheromone (\( \rho \) );
3. Numbers of ants (\( m \) )
4. the maximum generation (MAX)
5. The same starting value of pheromone (\( \tau_{ij} \) )
6. Give the pheromone to the optimal paths from the large amounts of paths random produced

**Step2** let all the ants at the first level, taboo table corresponding to the points where they poss.

**Step3** For every ant, it can select a point in with the transition probability \( P_{ij} \) , and the neighborhood of the selected point will subsume to the taboo table of the ant. Which can ensure the initial time of the trains have necessary safety interval.

**Step4** The solution can be got after every ant traversal all the n level. Then we have to judge whether the solution meets all constraints.
If the solution satisfy all the constraints, then calculate the value of the objective function \( f \).

And if this value better than the given value of path, see the updating of pheromone equation for the way of updating and make a new generation by \( k = k + 1 \).

If the solution can’t satisfy all the constraints, turn to step3.

**Step5** Distinguish whether the stop condition has been met, that is, whether the generation \( k \) reach the maximum generation \( MAX \),

If it is ,then stop operation and then output the best solution. That if \( \tau_{i,j} = \max_i \tau_{i,j} \), then it expresses that the initial time of jth train is \( a + t_0 - 1 \). Otherwise turn to step2.

3.5. The algorithm flow of heuristics ACO

We can get the flow of the heuristics ACO algorithm as follow(Fig 2.):

![Flow chart of ACO](image)

4. Empirical example

In the research, ACO algorithm was constructed. In order to validate its accuracy and efficiency of the application in solving the initial schedule of passenger trains problem, we
made an empirical investigation of 25 passenger trains, including 8 stations between city A and city B.

The effectiveness of optimization algorithm of initial schedule of passenger trains proposed in the paper is demonstrated by the examples by programming in Matlab7.0. With the ant colony algorithm simulation, while \( m = 30 \), \( MAX = 200 \), \( Q = 50 \), \( \tau_{ij} = 10 \), \( \rho = 0.8 \) and we give the pheromone to the optimal 40 paths from the 200 paths random produced. So we get the optimal solution is 21, which means the satisfaction of the passenger trains departure time. In order to analyze the effect of important factors and parameters to the solutions, we also get the following results.

The greater the number of ants, the better solution can be got. This is because with the increase in the number of ants, the search range becomes larger, but slow convergence and run time increase. The value of \( \tau_{ij} \) and \( Q \) can’t choose the too large number, because the objective function value is small. If they are too large, the influence of the pheromone updating equation will be lower.

5. Acknowledgment

This paper was prepared based on research project sponsored by Natural Science Foundation of China(No.60870004) and supported by the Fundamental Research Funds for the Central Universities(swjtu09zt19).

6. References


