Adaptive Fuzzy Tracking Control for MIMO Uncertain Nonlinear Time-delay Systems

Zhenbin Du, Tsung-Chih Lin
1 Yantai University, Yantai, 264005, China, zhenbindu@yahoo.com.cn
2 Feng-Chia University, Taichung, 40724, Taiwan, tclin@fcu.edu.tw
doi: 10.4156/ijact.vol3.issue6.2

Abstract
This paper addresses the problem of adaptive fuzzy tracking control for MIMO uncertain nonlinear time-delay systems. The control scheme combines adaptive fuzzy control with $\mathcal{H}_\infty$ control. Adaptive time-delay fuzzy logic systems are constructed and used to approximate the uncertain unknown time-delay functions. The $\mathcal{H}_\infty$ compensator is designed to eliminate fuzzy approximation errors and external disturbances. The adaptive laws for parameters are given by the tracking error. The time-delay Lyapunov function is constructed, and then it is proved that the error closed loop system satisfies the anticipant $\mathcal{H}_\infty$ tracking performance. Simulation results demonstrate that the control scheme is feasible.

Keywords: MIMO, Time-delay, Nonlinear Systems, Fuzzy Logic Systems, Tracking Control

1. Introduction

One of effective tools used to solve the problem of nonlinear control is adaptive fuzzy technique. Since adaptive fuzzy technique was proposed in [1], lots of extensions have been reported, see, e.g. [2-6], and references therein. Adaptive fuzzy technique has been proven to be a powerful tool for dealing with uncertainties in recursive structured systems. Adaptive fuzzy logic systems can uniformly approximate nonlinear continuous functions to arbitrary accuracy. The main advantage of adaptive fuzzy logic systems is that they can efficiently make use of the linguistic information and the expert information. The basic idea of adaptive fuzzy logic systems is to model uncertain nonlinear systems by a set of fuzzy “if-then” rules. When a proper control is given to the model, an anticipant output is appropriately produced from the nonlinear systems. Several adaptive fuzzy control schemes have been introduced to control SISO nonlinear systems [7-11], and the control schemes make nonlinear systems uniformly ultimately bounded. In [12-18], adaptive fuzzy controllers of MIMO nonlinear systems are designed to guarantee that the system outputs could track the anticipant signals. To further relax the conservatism of fuzzy modeling, type-2 fuzzy logic systems are proposed in [19]. Type-2 fuzzy logic systems have been also successfully used in nonlinear control [20-21]. Moreover, fuzzy method and intelligent methods are also combined and applied in engineering systems [22-23].

However, it is inevitable that many kinds of time-delay factors exist in practical engineering systems. Delay behavior may be caused by the measuring device, the measuring process, the control components and the actuator. For example, the robotic manipulator system has the delay behavior between the links. Time-delay also exists in electrical networks, turbojet engines and nuclear reactors, etc. Its existence is often a source of instability and poor performance. Therefore, the problem of stability for the nonlinear time-delay systems has attracted considerable attention. Based on adaptive technique, the papers [24-29] proposed several effective control schemes. In [24-25], authors presented two adaptive state feedback controllers of uncertain dynamical systems with the upper bound of time-delay functions. The adaptive fuzzy control was designed for a class of nonlinear time-delay systems with the correlated function in [26]. In [27-29], the control schemes combine fuzzy control and a backstepping approach. To further relax the conservatism of adaptive fuzzy modeling, an adaptive control scheme using type-2 fuzzy logic systems was developed to control SISO nonlinear time-delay systems in [30]. However, the approach requires a large number of weights, which results in unacceptably large learning time. The stability problem of MIMO nonlinear systems is more difficult because of the existence of coupling factors.
Motivated by these works, the goal of the paper is to present a new adaptive fuzzy tracking control scheme for a class of MIMO uncertain nonlinear time-delay systems. In the new method, adaptive time-delay fuzzy logic systems are constructed and used to approximate the uncertain unknown time-delay functions based on which a fuzzy controller is designed. The $H_{\infty}$ compensator is designed to eliminate fuzzy approximation errors and external disturbances. The adaptive laws for parameters are given by the tracking error. The error closed-loop system satisfies the antecedent $H_{\infty}$ tracking performance. Simulation results demonstrate that the control scheme is effective.

2. Problem formulation

Consider the following MIMO uncertain nonlinear time-delay systems

\[
\begin{align*}
\dot{x}_1 &= x_2, \\
\vdots \\
\dot{x}_{\beta_1 - 1} &= x_{\beta_1}, \\
\dot{x}_{\beta_i} &= f_i(x, x(t - \tau_1), \ldots, x(t - \tau_r)) + \sum_{i=1}^{m} g_{i}(x, x(t - \tau_1), \ldots, x(t - \tau_r))u_i + d_i, \\
\dot{x}_{\beta_{i+1}} &= x_{\beta_{i+2}}, \\
\vdots \\
\dot{x}_{\beta_n} &= f_m(x, x(t - \tau_1), \ldots, x(t - \tau_r)) + \sum_{i=1}^{m} g_{m}(x, x(t - \tau_1), \ldots, x(t - \tau_r))u_i + d_m, \\
y_1 &= x_1, \\
\vdots \\
y_m &= x_{(n - \beta_n + 1)}, \\
x(t) &= \Xi(t), t \in [-\varsigma, 0],
\end{align*}
\]

where $x$, $u$ and $y$ are the system state vector, control input vector and output vector, respectively; $x = [x_1, \ldots, x_{(\beta_1 - 1)}, \ldots, x_{(n - \beta_n + 1)}, \ldots, x_{(n - \beta_n + 1)}]^T \in \mathbb{R}^n$, $u = [u_1, \ldots, u_m]^T \in \mathbb{R}^m$, $y = [y_1, \ldots, y_m]^T \in \mathbb{R}^m$; $x$ is assumed to be measurable, $\beta_1 + \beta_2 + \cdots + \beta_n = n$, $f_i$, $g_i$ ($i, j = 1, \ldots, m$) are smooth functions; $d_i$ ($i = 1, \ldots, m$) denotes the external disturbance, $\Xi(t)$ is a continuous function, $\tau_i$ ($i = 1, \ldots, r$) denote time delays, $\varsigma = \max\{\tau_i | i = 1, \ldots, r\}$.

For convenience, define

\[
\begin{align*}
F(x, \tau) &= \Delta^T F(x, x(t - \tau_1), \ldots, x(t - \tau_r)) = [f_1(x, x(t - \tau_1), \ldots, x(t - \tau_r)), \ldots, f_m(x, x(t - \tau_1), \ldots, x(t - \tau_r))]^T, \\
G(x, \tau) &= \Delta^T G(x, x(t - \tau_1), \ldots, x(t - \tau_r)) = [G_1^T(x, x(t - \tau_1), \ldots, x(t - \tau_r)), \ldots, G_m^T(x, x(t - \tau_1), \ldots, x(t - \tau_r))]^T, \\
G_i(x, x(t - \tau_1), \ldots, x(t - \tau_r)) &= [g_{i1}(x, x(t - \tau_1), \ldots, x(t - \tau_r)), \ldots, g_{im}(x, x(t - \tau_1), \ldots, x(t - \tau_r))]^T \\
(i = 1, \ldots, m).
\end{align*}
\]

Then the nonlinear system (1) is rearranged as

\[
\begin{align*}
\dot{x} &= Ax + B[F(x, \tau) + G(x, \tau)u + d], \\
y &= Cx, \\
x &= \Xi(t), t \in [-\varsigma, 0],
\end{align*}
\]

- 11 -
where $\mathbf{d} = \text{diag}[A_1, \cdots, A_m], B = \text{diag}[B_1, \cdots, B_m]$ and $C = \text{diag}[C_1, \cdots, C_m]$,

$$
A_i = \begin{bmatrix}
0 & 1 & 0 & \cdots & 0 \\
0 & 0 & \ddots & \ddots & \vdots \\
\vdots & \ddots & \ddots & 0 & 0 \\
0 & 1 & 0 & \cdots & 0
\end{bmatrix} \in \mathbb{R}^{\beta_i \times \beta_i},
$$

$$
B_i = [0, \cdots, 0, 1]^T, C_i = [1, 0, \cdots, 0]_{(x \beta_i)}, F(x, \tau) \text{ and } G(x, \tau) \text{ are uncertain unknown parts.}
$$

For the given reference signals $y_{r1}, \cdots, y_{rm}$, define the tracking errors as

$$
e_i = y_{r1} - y_1, \cdots, e_m = y_{rm} - y_m.$$

Denote $y_{r} = [y_{r1}, \cdots, y_{rm}], y_{r}^{(\beta)} = [y_{r1}^{(\beta_1)}, \cdots, y_{rm}^{(\beta_m)}],

\begin{equation}
Y_m = [y_{r1}, \cdots, y_{r1}^{(\beta_1 - 1)}, \cdots, y_{rm}, \cdots, y_{rm}^{(\beta_m - 1)}]^T,
\end{equation}

and the function $y_{r}^{(\beta)}$ is membership function.

### Control objectives:

Determine a feedback controller and adaptive laws of the parameters such that the error closed loop system satisfies the anticipant $H_\infty$ tracking performance. Meanwhile, the outputs of the system could fast track the given reference signals.

**Assumption:** $\forall x \in U_x, U_x$ is a compact set, the matrix $G(x, \tau)$ is nonsingular.

### 3. Adaptive time-delay fuzzy logic systems

Employing the universal approximation property of fuzzy logic systems for nonlinear continuous functions, we construct adaptive time-delay fuzzy logic systems to approximate the uncertain unknown time-delay functions. The approximation for the $k$th element of the vector function $F(x, \tau)$ is of the following form:

$$
R^l : \text{If } x_1 \in A_1^l, \cdots, x_n \in A_n^l, \text{ Then } y \in G^l \ (l = 1, \cdots, p)
$$

$$
\hat{F}_k(x, \tau | \theta) = \xi_k^T (x, \tau) \theta_k,
$$

Where

$$
\xi_k^T (x, \tau) = (\xi_1^k(x, \tau), \cdots, \xi_p^k(x, \tau)) \in \mathbb{R}^p,
$$

$$
\xi_i^k = \prod_{l=1}^n \mu_F^j(x_1, \tau) \prod_{l=1}^n \mu_F^j(x_2, \tau) \cdots \prod_{l=1}^n \mu_F^j(x_n, \tau),
$$

$$
\mu_F^j(x_1, \tau) = \mu_F^j(x_1) \prod_{l=1}^{\tau} \mu_F^j(x_1) (t - \tau_l) ,
$$

$$
\theta_k = (\theta_1^k, \cdots, \theta_p^k)^T \in \mathbb{R}^p,
$$

and the function $\mu_F^j(\cdot)$ is membership function.
Based on (3), we give the adaptive time-delay fuzzy logic systems for the time-delay vector function $F(x,\tau)$ and the time-delay matrix function $G(x,\tau)$ as follows:

\begin{align}
\hat{F}(x,\tau | \Theta_1) = \Psi(x,\tau)\Theta_1, \quad \hat{G}(x,\tau | \Theta_2) = \Psi(x,\tau)\Theta_2
\end{align}

where $\Psi(x,\tau) = \text{diag}[\Sigma(x,\tau), \cdots, \Sigma_m(x,\tau)]$, weights $\Theta_1, \Theta_2$ are adjustable parameters.

Define the estimation errors of the parameters

\[ \tilde{\Theta}_1 = \Theta_1 - \Theta_1^*, \quad \tilde{\Theta}_2 = \Theta_2 - \Theta_2^*, \]

then the approximation errors of adaptive time-delay fuzzy logic systems for the time-delay vector function $F(x,\tau)$ and the time-delay matrix function $G(x,\tau)$ can be expressed as

\begin{align}
\hat{F}(x,\tau | \Theta_1, \alpha, \delta) - F(x,\tau) = \Psi(x,\tau)\tilde{\Theta}_1 + w_1, \quad (5) \\
\hat{G}(x,\tau | \Theta_2, \alpha, \delta) - G(x,\tau) = \Psi(x,\tau)\tilde{\Theta}_2 + w_2, \quad (6)
\end{align}

where $w_1, w_2$ are the residual terms.

4. Controller design

A fuzzy controller is chosen as

\[ u = \hat{G}(x,\tau | \Theta_2)^{-1}[\hat{F}(x,\tau | \Theta_1) + y_f^f + K^Te - u_{\text{con}}], \quad (7) \]

where $\hat{F}(x,\tau | \Theta_1)$ and $\hat{G}(x,\tau | \Theta_2)$ are given by (4), $K^T$ is the feedback gain matrix, which makes the characteristic polynomial of $A - BK^T$ Hurwitz, $u_{\text{con}}$ is a compensator to attenuate the approximation errors and the external disturbances.

Applying (7) to (2), we obtain the tracking error dynamic equation

\[ \dot{e} = Ae - BK^T \dot{e} + B((\hat{F}(x,\tau | \Theta_1) - F(x,\tau)) + (\hat{G}(x,\tau | \Theta_2) - G(x,\tau))u - d) + Bu_{\text{con}}. \quad (8) \]

Define the optimal parameters

\begin{align}
\Theta_1^T = \min_{\Theta_1 \in \Omega_1} \sup_{x \in \mathcal{U}_1} \| \hat{F}(x,\tau | \Theta_1) - F(x,\tau) \|, \\
\Theta_2^T = \min_{\Theta_2 \in \Omega_2} \sup_{x \in \mathcal{U}_1} \| \hat{G}(x,\tau | \Theta_2) - G(x,\tau) \|
\end{align}

where $\mathcal{U}_1 = \{x \in \mathbb{R}^n\}$, $\Omega_1 = \{\Theta_1 \in \mathbb{R}^{m \times 1}\}$, $\Omega_2 = \{\Theta_2 \in \mathbb{R}^{m \times m}\}$, $\mathcal{U}_1$, $\Omega_1$, $\Omega_2$ denote the sets of suitable bounds on $x, \Theta_1, \Theta_2$, respectively.

The optimal approximation error is defined as $\overline{w} = w - d$. Using (5), (6), the error dynamic equation of the observer is rewritten as

\[ \dot{e} = (A - BK^T) e + B(\Psi(x,\tau)\tilde{\Theta}_1 + \Psi(x,\tau)\tilde{\Theta}_2)u + B\overline{w} + Bu_{\text{con}}. \quad (11) \]
where \( w = w_1 + w_2 u \), \( \hat{\Theta}_1 = \Theta_1 - \Theta_1^* \), \( \hat{\Theta}_2 = \Theta_2 - \Theta_2^* \).

Choosing the parameter adaptive laws by use of the tracking error

\[
\dot{\hat{\Theta}}_1 = -\eta_1 \Psi^T(x, t)B^T P e,
\]
\[
\dot{\hat{\Theta}}_2 = -\eta_2 \Psi^T(x, t)B^T Pe u^T,
\]

where \( \eta_1 \) and \( \eta_2 \) are positive constants.

The compensator

\[
u_{com} = -(1/\alpha)B^T P e.
\]

The symmetric and positive definite matrix \( P \) could be got by the following Riccati equation

\[
(A - BK^T)^T P + P(A - BK^T) + Q - (2/\alpha - 1/\rho^2)PBB^T P = 0,
\]

where \( 2\rho^2 \geq \alpha > 0 \), \( Q \) is a symmetric and positive definite matrix.

**Theorem 1:** For the nonlinear system (1), the adaptive fuzzy controller is chosen as (8) with the time-delay fuzzy logic systems (4), the parameter adaptive laws (12) and (13) and the compensator (14), then the error closed loop system satisfies \( H^\infty \) tracking performance

\[
\int_0^T e^T Q e dt \leq e^T(0)Pe(0) + \sum_{i=1}^r \int_{t_i - \tau_i}^{t_i} \Xi^T(\nu)\Xi(\nu)dv + \frac{1}{\eta_1} \hat{\Theta}_1^T(0)\hat{\Theta}_1(0) + \frac{1}{\eta_2} \text{tr}(\hat{\Theta}_2^T(0)\hat{\Theta}_2(0))
\]
\[
+ \rho^2 \int_0^T (\hat{w}^T \hat{w}) dt,
\]

where \( \hat{\Theta} = Q - \alpha I > 0 \).

**Proof:** The time-delay Lyapunov function is selected as

\[
V = \frac{1}{2} e^T P e + \frac{1}{2} \sum_{i=1}^r \int_{t_i - \tau_i}^{t_i} e^T(\nu)e(\nu)dv + \frac{1}{2\eta_1} \hat{\Theta}_1^T(0)\hat{\Theta}_1(0) + \frac{1}{2\eta_2} \text{tr}(\hat{\Theta}_2^T(0)\hat{\Theta}_2(0)).
\]

Obviously \( V > 0 \).

\[
\dot{V} = \frac{1}{2} e^T [(A - BK^T)^T P + P(A - BK^T)]e + e^T PBu_{com} + e^T PBw + \frac{1}{2} \sum_{i=1}^r e^T(t)e(t)
\]
\[-\frac{1}{2} \sum_{i=1}^r e^T(t - \tau_i)e(t - \tau_i) + e^T \Psi^T(x, \tau)\hat{\Theta}_1 + \frac{1}{\eta_1} \hat{\Theta}_1^T\hat{\Theta}_1 + e^T \Psi^T(x, \tau)\hat{\Theta}_2 + \frac{1}{\eta_2} \text{tr}(\hat{\Theta}_2^T\hat{\Theta}_2),
\]

Noting that \( \dot{\Theta}_1 = \dot{\hat{\Theta}}_1 \), \( \dot{\Theta}_2 = \dot{\hat{\Theta}}_2 \), we can rearrange (16) by use of (12) and (13)
\[
\dot{V} \leq \frac{1}{2} e^T [(A - BK^T)^T P + P(A - BK^T) + rI] e + e^T PB_\text{com} + e^T PB\overline{w}
\]

\[
= \frac{1}{2} e^T [(A - BK^T)^T P + P(A - BK^T) - \frac{2}{\alpha} PBB^T P + rI] e + e^T PB\overline{w}
\]

\[
= -\frac{1}{2} e^T (Q - rI)e - \frac{1}{2\rho^2} e^T PB\overline{w}^T Pe + e^T PB\overline{w}
\]

\[
= -\frac{1}{2} e^T (Q - rI)e - \frac{1}{2\rho} (\frac{1}{\rho} B^T Pe - \rho\overline{w})^T (\frac{1}{\rho} B^T Pe - \rho\overline{w}) + \frac{1}{2\rho^2} \overline{w}^T \overline{w}
\]

\[
\leq -\frac{1}{2} e^T (Q - rI)e + \frac{1}{2\rho^2} \overline{w}^T \overline{w}
\]

Let \( \overline{Q} = Q - rI \), then

\[
\dot{V} \leq -\frac{1}{2} e^T \overline{Q} e + \frac{1}{2\rho^2} \overline{w}^T \overline{w} \leq -\frac{1}{2} \lambda_{\text{min}}(\overline{Q}) \| e \|^2 + \frac{1}{2\rho^2} \| \omega \|^2 .
\]  (17)

When \( \| e \| > \frac{\rho}{\lambda_{\text{min}}(\overline{Q})} \| \overline{w} \| \), \( \dot{V} < 0 \). Then the error closed-loop system is stable.

Integrating (17) in \([0, T]\) yields

\[
\int_0^T e^T \overline{Q} e dt \leq e^T(0)Pe(0) + \sum_{i=1}^r \int_{-\tau_i}^0 \Xi^T(\nu)\Xi(\nu) d\nu + \frac{1}{\eta_1} \Theta_1^T(0)\Theta_1(0) + \frac{1}{\eta_2} \text{tr}(\Theta_2^T(0)\Theta_2(0))
\]

\[
+ \rho^2 \int_0^T (\overline{w}^T \overline{w}) dt .
\]

5. Simulation example

Consider the 2-link manipulator in [25]

\[
\ddot{q}(t) + C(q, \dot{q}) \dot{q}(t) + g(q) = B(q)\Gamma(t) + \sum_{i=1}^r \xi_i(t)q(t - \tau_i) + d',
\]

\[
q(t) = \Xi(t), t \in [-\varepsilon, 0],
\]

where \( C(q, \dot{q}) = H^{-1}(q)C(q, \dot{q}), g(q) = 0, B(q) = H^{-1}(q), q = [q_1, q_2]^T, \xi_i(t)(i = 1, \cdots, r) \) are assumed to be bounded. \( \tau_i (i = 1, \cdots, r) \) denote time delays, \( d' = H^{-1}(q)d \) is the external disturbance, \( d \) is random noise with zero mean and variance 0.05, and \( d \) is bounded. The detailed date are in [31].

Denote \( x_1 = q_1, x_2 = \dot{q}_1, x_3 = q_2, x_4 = \dot{q}_2, u_1 = \Gamma_1, u_2 = \Gamma_2, y_1 = x_1, y_2 = x_3 \). MIMO
uncertain nonlinear time-delay system (18) is rearranged as
\[ \dot{x} = Ax + B[F(x, \tau) + G(x, \tau)u + d], \]
\[ y = Cx, \]
\[ x = \Xi(t), t \in [-\tau, 0], \]
where \( F(x, \tau) = -H^{-1}C + \sum_{i=1}^{r} \xi_i(t)[x_1(t-\tau_i) \quad x_3(t-\tau_i)]^T \), \( G(x, \tau) = H^{-1} \), \( A = \text{diag}(A_1, A_2) \),
\[ B = \text{diag}(B_1, B_2), \quad C = \text{diag}(C_1, C_2), \quad A_1 = A_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad B_1 = B_2 = \begin{bmatrix} 0 & 1 \\ \end{bmatrix}^T, \quad C_1 = C_2 = \begin{bmatrix} 1 & 0 \end{bmatrix}, \]
\( r = 2, \tau_1 = 0.1, \tau_2 = 0.05, \Xi(t) = [0.1 \quad 0.05 \quad 0.1 \quad 0.05] \).

Choose
\[ K = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 2 & 2 \end{bmatrix}, \]
In (12-13), the parameters \( \eta_1 = 0.2, \eta_2 = 0.2 \), and the symmetric and positive definite matrix
\[ P = \begin{bmatrix} 0.3018 & 0.3064 \\ 0.3064 & 0.3159 \\ 10.2926 & 0.2969 \\ 0.2969 & 0.3061 \end{bmatrix}. \]

Seven fuzzy rules are defined in adaptive fuzzy logic systems.
\[ R^{(j)}: \text{if } x_1 \text{ is } F_1^{(j)}, \ldots, \quad x_4 \text{ is } F_4^{(j)}, \text{ then } y \text{ is } G^{(j)} \quad (j = 1,2,\cdots,7), \]
\[ \mu_{F_1^{(j)}}(x_i) = \frac{1}{1 + \exp[5(x_i + 0.8)]} \quad (i = 1,2,\cdots,4), \]
\[ \mu_{F_2^{(j)}}(x_i) = \exp[-(x_i + 0.6)^2] \quad (i = 1,2,\cdots,4), \]
\[ \mu_{F_3^{(j)}}(x_i) = \exp[-(x_i + 0.4)^2] \quad (i = 1,2,\cdots,4), \]
\[ \mu_{F_4^{(j)}}(x_i) = \exp[-(x_i)^2] \quad (i = 1,2,\cdots,4), \]
\[ \mu_{G_1^{(j)}}(x_i) = \exp[-(x_i - 0.6)^2] \quad (i = 1,2,\cdots,4), \]
\[ \mu_{G_2^{(j)}}(x_i) = \exp[-(x_i - 0.4)^2] \quad (i = 1,2,\cdots,4), \]
\[ \mu_{G_3^{(j)}}(x_i) = \exp[-(x_i - 0.6)^2] \quad (i = 1,2,\cdots,4), \]
Adaptive Fuzzy Tracking Control for MIMO Uncertain Nonlinear Time-delay Systems
Zhenbin Du, Tsung-Chih Lin
International Journal of Advancements in Computing Technology Volume 3, Number 6, July 2011

\[ \mu_{F_i}^*(x_i) = \frac{1}{1 + \exp[5(x_i - 0.8)]} \quad (i = 1,2,\cdots,4). \]

Denote \( S_i = \sum_{j=1}^{7} \prod_{k=1}^{4} \mu_{F_{i,j}}(x_i,\tau) \) with \( \mu_{F_{i,j}}(x_i,\tau) = \mu_{F_{i,j}}(x_i) \prod_{k=1}^{2} \mu_{F_{i,k}}(x_i(t - \tau_k)) \), then

\[ \xi(x,\tau) = [\prod_{j=1}^{4} \mu_{F_{i,j}}(x_i,\tau) / S_i_1, \cdots, \prod_{j=1}^{4} \mu_{F_{i,j}}(x_i,\tau) / S_i_7] = [\xi_1,\cdots,\xi_7]. \]

\[ \Psi(x,\tau) = \text{diag} [\xi^T(x,\tau), \xi^T(x,\tau)]. \]

Using the proposed method in the paper, we design the fuzzy controller to track the signals \( y_{r1} = (\pi / 30) \sin(2\tau) \), \( y_{r2} = (\pi / 30) \cos(2\tau) \). Simulation results are given in figure 1 and figure 2. Figure 3 and figure 4 are the input curves.

**Figure 1.** \( y_1 \) (dashed line), \( y_{r1} \) (solid line)

**Figure 2.** \( y_2 \) (dashed line), \( y_{r2} \) (solid line)
6. Conclusion

This paper presents a new adaptive fuzzy scheme for MIMO uncertain nonlinear time-delay systems. Adaptive time-delay fuzzy logic systems are implemented to model the nonlinear systems. Theory analysis verifies the feasibility of the proposed control scheme and simulation results demonstrate the effectiveness of the proposed control scheme.

7. Acknowledgements

This work was supported by the National Natural Science Foundation of China (60974028).

8. References

Adaptive Fuzzy Tracking Control for MIMO Uncertain Nonlinear Time-delay Systems
Zhenbin Du, Tsung-Chih Lin
International Journal of Advancements in Computing Technology Volume 3, Number 6, July 2011


Adaptive Fuzzy Tracking Control for MIMO Uncertain Nonlinear Time-delay Systems
Zhenbin Du, Tsung-Chih Lin
International Journal of Advancements in Computing Technology Volume 3, Number 6, July 2011