Distributed Localization Algorithm Based on Statistical Uncorrelated Vectors

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Abstract

We have developed a new localization algorithm based on a set of uncorrelated discriminant vectors (SUV). Comparing to the centralized multidimensional localization algorithm MDS-MAP that has been widely used in wireless sensor networks, this algorithm can improve the localization accuracy and reduce the computing complexity. In this algorithm, the solving equation of the double centered matrix is simplified by coordinate transformation. Then a new double centered matrix is reconstructed using a set of uncorrelated discriminant vectors in order to reduce the localization error caused by the ranging error. The direct calculation of node coordinates, as well as the distributed and incremental localization, can be realized. Simulation results indicate that after incorporation of our algorithm, the range of the localization error variation decreases rapidly and the localization accuracy becomes more stable when the number of anchors increases. In the case of large ranging error, the localization accuracy of the new algorithm is increased by about 50.16% and 62.24% than that of the trilateration location method and the MDS-MAP method, respectively.

Keywords: Wireless Sensor Network, Ranging Error, Statistical Uncorrelated Vectors, Localization Algorithm

1. Introduction

The basic function of wireless sensor network (WSN) [1] is to connect a series of spatial dispersion of sensor nodes through self-organizing wireless network, transfer and summarize the collected data, and carry out the appropriate analysis and processing. So it can realize the collaborative monitoring and controlling of the physical or environmental situations in a large space. Typically, most sensor nodes in WSN are randomly distributed in space. If the sensor unit does not know its own location, the data collected will not make sense in most practical applications. As an additional motivation, wireless sensor nodes location information can be extremely useful for scalable, geographic routing algorithms. Note that location itself is often the data that needs to be sensed. Therefore, the localization of sensor nodes in WSN is very important [2].

Currently proposed localization algorithms can be divided into different categories according to different standards. For example, localization algorithms can be classified into range-based and range-free ones according to whether the range measuring is required or not. Range-based localization algorithms usually use the measured distance or angle information between nodes to compute the node position. The trilateration location, triangulation location, and maximum likelihood estimation algorithms [3] are typical range-based algorithms. Range-free localization algorithms are basically based on the network connectivity and other information to realize the positioning of unknown nodes. The DV-position [4-6], convex position estimation [7] and multidimensional scaling map (MDS-MAP) [8,9] are well-known range-free algorithms. Among them, MDS-MAP can also be used for range-based positioning. Typical ranging techniques include Received Signal Strength Indicator (RSSI) [10,11], Time of Arrival (TOA) [12], Time Difference of Arrival (TDOA) [13] and Angle of Arrival (AOA) [14]. The RSSI based technique has the advantages of high speed and low cost, but it is easy to be affected by the environment. Thus, this method usually needs an environment information database. For range-based localization algorithms,
multiple ranging, loop positioning, error compensation and other appropriate methods can be adopted to further improve the positioning accuracy.

MDS-MAP [8,9] originates from a data-analysis technique by displaying distance-like data in geometrical visualization. It computes the pair-wise distance between all pairs of nodes to build a distance matrix, and constructs the double centered matrix with the distance matrix. Then, it applies the classical Multidimensional Scaling (MDS) to the double centered matrix, to retain the first two largest eigenvalues and eigenvectors to a 2-dimensional (2D) relative map. After that, with three given anchors, the relative map can be transformed into an absolute map based on the anchors’ absolute locations. However, the deep analysis of the classical MDS-MAP algorithm indicates that it has several disadvantages in practical applications. First, it is a centralized location algorithm, so it is difficult to be used in distributed positioning applications. Second, it calculates the relative coordinates between the nodes and gets the absolute coordinates by appropriate scaling, rotation and reflection. The calculation is very time-consuming, and it is easy to introduce new errors. Third, it only retains the first two largest eigenvalues and eigenvectors to a 2D relative map. When the ranging error is large, this approach may cause rapid decrease of the localization accuracy. Finally, when the number of nodes changes, it must recalculate the double centering matrix, resulting in a significant increase in computation. To solve the above problems, this paper introduces a new distributed localization algorithm based on uncorrelated discriminant vectors, and presents its incremental localization algorithm. The solving equation of the double centered matrix can be simplified by the coordinate transformation. In order to reduce the noise disturbance, a new double centered matrix is reconstructed using a set of uncorrelated discriminant vectors, which can be used to calculate the node coordinates directly.

2. MDS-MAP Localization Algorithm

The MDS-MAP localization algorithm assumes that nodes are distributed in the m-dimensional space \( m \) is generally taken to be 2 or 3. Each node is designated an identity number in the network. Let the coordinates of the node \( i \) be \( x_i = (x_{i1}, x_{i2}, \ldots, x_{im}) \), if there are \( n \) nodes in the network, the coordinate matrix of all nodes is \( X = (x_1, x_2, \ldots, x_n)^T \), the pair-wise distance matrix of all nodes is \( D_n = (d_{ij})_{n \times n} \), and the Euclidean distance between \( x_i \) and \( x_j \) in an \( m \)-dimensional space is

\[
d_{ij} = \sqrt{\sum_{k=1}^{m} (x_{ik} - x_{jk})^2}.
\]

The distance matrix \( D_n \) is related to the double centered matrix \( B_n = (b_{ij})_{n \times n} = \bar{X}_n \bar{X}_n^T \) by the following transformation:

\[
b_{ij} = \sum_{k=1}^{m} x_{ik} x_{jk} = -\frac{1}{2} \left( d_{ij}^2 - \frac{1}{n} \sum_{j=1}^{n} d_{ij}^2 - \frac{1}{n} \sum_{i=1}^{n} d_{ij}^2 + \frac{1}{n^2} \sum_{j=1}^{n} \sum_{j=1}^{n} d_{ij}^2 \right),
\]

where \( \bar{X}_n \) is the relative coordinates of the nodes.

For a real symmetric matrix \( B_n \), there exists an orthogonal matrix \( V \):

\[
B_n = V A V^T = V A^{1/2} (A^{1/2} V),
\]

where \( A \) is a diagonal matrix, and \( \bar{X}_n = V A^{1/2} \). Thus, if the distance matrix \( D_n \) is known, relative coordinates of nodes can be calculated. Finally, given sufficient anchor nodes (3 or more for 2D, 4 or more for 3D), the relative map can be transformed to an absolute map based on the absolute positions of anchors. The transformations include operations of scaling, rotation, and reflection.

3. SUV Localization Algorithm

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In this section, we present a detailed derivation of the distributed localization algorithm based on a set of uncorrelated discriminant vectors, and give its incremental algorithm. We assume that nodes are distributed in the \( m \)-dimensional space (\( m \) is generally taken to be 2 or 3), and each node is designated an identity number in the network. Let the coordinates of node \( i \) be \( x_i = (x_{i1}, x_{i2}, \ldots, x_{im}) \), if there are \( k \) (\( k \geq 3 \)) anchor nodes in the network, the coordinate matrix of all anchor nodes is \( X' = (x'_{1}, x'_{2}, \ldots, x'_{k})^T \). Shifting all anchor nodes so that if \( x'_{1} \) is in the origin of coordinates, then \( x_{1} \) is the new coordinates of \( x'_{1} \). The shift value is \( a = (a_{1}, a_{2}, \ldots, a_{m}) \), and \( x_{1} \) can be any anchor node in the network. After the shifting, the coordinate matrix of all anchor node is \( X_k = (x_{1}, x_{2}, \ldots, x_{k})^T \).

The pair-wise distance matrix of all anchor nodes is \( D_k = (d_{ij})_{k \times k} \), and the Euclidean distance between two anchor nodes \( x_i \) and \( x_j \) in an \( m \)-dimensional space is

\[
d_{ij} = \sqrt{\sum_{r=1}^{m} (x_{ir} - x_{jr})^2}.
\]

(4)

The distance matrix \( D_k \) is related to the double centered matrix \( B_k = (b_{ij})_{k \times k} = \tilde{X}_k \tilde{X}_k^T \) by the follow transformation:

\[
b_{ij} = (d_{ii}^2 + d_{jj}^2 - d_{ij}^2) / 2.
\]

(5)

In the distributed positioning, the parameters \( a, D_k, \) and \( B_k \) can be stored into the unknown node, or sent to the unknown node by the wireless signal. For any unknown node \( k+1 \) in the network, the pair-wise distances \( d_{i(k+1)}, d_{2(k+1)}, \ldots, d_{k(k+1)} \) between the unknown node and all anchor nodes can be obtained using the ranging techniques such as RSSI.

Since \( b_{ij} = b_{ji} \), the double centered matrix \( B_{k+1} \) can be express as

\[
B_{k+1} = \begin{bmatrix}
B_k & b_{k,(k+1)} \\
b_{1,(k+1)} & \cdots & b_{k,(k+1)} & b_{(k+1),(k+1)}
\end{bmatrix}.
\]

(6)

The matrix \( B_{k+1} \) is not always nonsingular, especially for the localization in WSN. Thus, there is at least one zero eigenvalue of \( B_{k+1} \). We can rewrite the eigen decomposition of \( B_{k+1} \) as

\[
B_{k+1} = (V_1, V_2) \begin{pmatrix}
D_1 & 0 \\
0 & D_2
\end{pmatrix} \begin{pmatrix}
V_1^T \\
V_2^T
\end{pmatrix},
\]

(7)

where \( D_1 \) is the \( r \times r \) diagonal matrix containing \( r \) eigenvalues greater than zero, \( V_1 \) contains the \( r \) eigenvectors corresponding to the \( r \) eigenvalues of \( D_1 \), \( D_2 \) is the \( (k+1-r) \times (k+1-r) \) diagonal matrix containing the other \( k+1-r \) eigenvalues, \( V_2 \) contains the \( k+1-r \) eigenvectors corresponding to the other \( k+1-r \) eigenvalues of \( D_2 \).

The matrix \( B_{k+1} \) is reconstructed by the following equation:

\[
B_{k+1} = V_1 D_1 V_1^T.
\]

(8)
According to $B_k = (b_{ij})_{k \times k} = X_k X_k^T$, we have

$$X_k x_{(k+1)}^T = b_{(k+1)}.'$$

The unknown node coordinate $x_{(k+1)}^T$ can then be calculated from

$$x_{(k+1)}^T = (X_k^T X_k)^{-1} X_k^T b_{(k+1)}.'$$

where $b_{(k+1)}.' = (b_{1,(k+1)}, b_{2,(k+1)}, ..., b_{(k+1),(k+1)})^T$. Finally, the unknown node coordinate can be obtained from

$$x_{(k+1)} = x_{(k+1)} - a.'$$

According to the above derivation, the new localization algorithm based on uncorrelated discriminant vectors includes the following steps:

1) Shift the coordinates of the anchor node so that one anchor node is in the origin of coordinates.
2) Measure the pair-wise distances between the unknown node and all anchor nodes using the ranging techniques, and calculate the distance matrix and the double centered matrix.
3) Decompose and reconstruct the double centered matrix.
4) Calculate the coordinate of the unknown node and shift it to the original coordinate system.

The incremental localization algorithm first locates the unknown nodes near the anchor nodes, then turns the located unknown nodes into the anchor nodes until the positioning of all unknown nodes is completed. According to the equation (6), the double centered matrix can be used iteratively, so the proposed algorithm is more convenient to realize incremental positioning.

Assuming that there are $q$ anchor nodes and $p-q$ unknown nodes in the network, $X_q$ and $D_q$ are known, and the first anchor node is at the coordinate origin. So the incremental localization algorithm based on a set of uncorrelated discriminant vectors can be expressed by:

a) The first $q+1$ unknown node receives $X_q$, $D_q$ and $B_q$ from anchor nodes.

b) Measure the pair-wise distances $d_{1,(q+1)}, d_{2,(q+1)}, ..., d_{(q+1), (q+1)}$ between the unknown node and the anchor nodes using the ranging techniques, and calculate the double centered matrix $B_{q+1}$.

c) Calculate the coordinate $x_{q+1}$ of the $q+1$ unknown node by the above method.

d) Reconstruct $X_{q+1}$, $D_{q+1}$ and $B_{q+1}$, and send them to the $q+2$ unknown node.

e) Repeat steps a)-d) until the coordinates of all the $p-q$ unknown nodes are calculated.

The disadvantages of the above incremental localization algorithm are, when the number of unknown nodes increases, the computation will be greatly increased and the ranging error may accumulate and spread during the localization process.

4. Experiment

To evaluate the performance of the proposed algorithm, we perform simulations using Matlab. The sensor nodes are randomly deployed in a 2D square region. We compare our method with the trilateration location method and the MDS-MAP method. For simplicity, we assume that the measured pair-wise distance is

$$\tilde{d}(x_i, x_j) = d(x_i, x_j)(1 + N(0, \sigma^2)),$$

where $d(x_i, x_j)$ is the real distance, and $N(0, \sigma^2)$ denotes an independently generated normal random variable with the standard deviation $\sigma$.

The performance of the algorithms can be evaluated by the average location estimation error, which is defined as
\[
error = \frac{\sum_{i=q+1}^{p} \| \vec{x}_i - \vec{x}_q \|_2}{p-q},
\]

where \(\| \cdot \|_2\) denotes the Euclidean distance. Usually, the localisation algorithm has better performance if the error is lower.

In the first experiment, ninety unknown nodes are randomly deployed in a square region of 100-by-100 m\(^2\), and ten anchors are also randomly deployed. Fig.1 shows the final positioning result of using the SUV algorithm when fixing \(\sigma=0.2\) (namely the ranging error is 20\%). The square symbols represent the locations of anchor nodes, the circles represent the true locations of unknown nodes, and the solid lines represent the deviations of the calculated positions from the true positions that increase with the line length. The average localization error in this experiment is about 5.51 m. Under the same conditions, the average localization error obtained by using the trilateration and MDS-MAP methods are 9.72 m and 8.87 m, respectively. These results suggest that the proposed localization algorithm can improve the localization accuracy efficiently when the ranging error is relatively large.

![Figure 1](image.png)

**Figure 1** Positioning results using the proposed SUV localization algorithm. The squares represent the locations of anchor nodes, the circles represent the true locations of unknown nodes, and the solid lines represent the deviations of the calculated position from the true position.

In the second experiment, we study the relationship between the localization error and the number of anchor nodes when fixing the ranging error to be 20\%. The positioning results of three different localization algorithms are presented in Table 1. The localization errors of the trilateration and MDS-MAP methods are relatively large when the number of anchor nodes is less than 5. When the number of anchor nodes is larger than 5, the localization errors of these two algorithms begin to be stable. In contrast, the localization error of the proposed SUV localization algorithm is always less than the other two algorithms, and more importantly, the variation of the localization error also decreases with increasing number of anchor nodes, suggesting that the localization results of the proposed algorithm are more accurate and stable.

**Table 1** Relationship between the number of anchors and the localization error using three different localization algorithms

<table>
<thead>
<tr>
<th>Number of anchors</th>
<th>Localization error (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Trilateration</td>
</tr>
<tr>
<td>3</td>
<td>16.87±3.88</td>
</tr>
</tbody>
</table>
In the third experiment, we study the relationship between the localization error and the ranging error when fixing the number of the anchor nodes to be 6. The positioning results of three localization algorithms are presented in Table 2. The results indicate that the proposed algorithm has better anti-interference characteristics than the other algorithms when the ranging error increases from 5% to 60%. For example, the localization errors of the trilateration and MDS-MAP methods increase rapidly when the ranging error is more than 20%. In contrast, the average localization error of the proposed algorithm is smaller, approximately 50.16% and 62.24% less than that of the trilateration and MDS-MAP methods when the ranging error is from 20% to 60%, respectively.

<table>
<thead>
<tr>
<th>Ranging error (%)</th>
<th>Trilateration Localization error (m)</th>
<th>MDS-MAP</th>
<th>SUV</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>2.66±0.31</td>
<td>1.48±0.12</td>
<td>2.07±0.33</td>
</tr>
<tr>
<td>10</td>
<td>5.62±0.68</td>
<td>2.46±0.83</td>
<td>3.29±0.76</td>
</tr>
<tr>
<td>15</td>
<td>8.14±2.13</td>
<td>5.16±1.41</td>
<td>5.23±1.02</td>
</tr>
<tr>
<td>20</td>
<td>9.44±1.98</td>
<td>8.31±1.83</td>
<td>5.90±1.52</td>
</tr>
<tr>
<td>25</td>
<td>12.63±3.12</td>
<td>23.93±4.79</td>
<td>6.73±1.37</td>
</tr>
<tr>
<td>30</td>
<td>16.08±4.56</td>
<td>36.07±5.13</td>
<td>7.15±2.03</td>
</tr>
<tr>
<td>35</td>
<td>17.97±3.87</td>
<td>38.02±5.85</td>
<td>9.41±1.98</td>
</tr>
<tr>
<td>40</td>
<td>25.02±3.54</td>
<td>35.32±6.16</td>
<td>12.55±2.56</td>
</tr>
<tr>
<td>45</td>
<td>28.49±5.90</td>
<td>36.51±5.97</td>
<td>12.84±2.17</td>
</tr>
<tr>
<td>50</td>
<td>28.65±5.35</td>
<td>36.09±6.56</td>
<td>16.38±2.89</td>
</tr>
<tr>
<td>55</td>
<td>35.16±4.97</td>
<td>39.31±7.12</td>
<td>16.30±2.62</td>
</tr>
<tr>
<td>60</td>
<td>44.56±6.15</td>
<td>42.91±7.53</td>
<td>16.56±2.46</td>
</tr>
</tbody>
</table>

5. Conclusion

In this paper, we propose a new localization algorithm based on a set of uncorrelated discriminant vectors and give its incremental localization algorithm. The solving equation of the double centered matrix is simplified by the coordinate transformation. In order to reduce the noise disturbance, a new double centered matrix is reconstructed using a set of uncorrelated discriminant vectors, which can be used to calculate the node coordinates directly. Simulation results indicate that the proposed localization algorithm can improve the localization accuracy efficiently when the ranging error is relatively large, which is particularly suitable for positioning sensor nodes in wireless sensor networks constructed on the basis of low cost hardware.

6. Acknowledgment

This work has been supported by SRF for ROCS, SEM, the Foundation of Key Laboratory of Advanced Process Control for Light Industry (APCLI1001), Ministry of Education, and the Fundamental Research Funds for the Central Universities of China (JUSRP20914, JUDCF10031).

7. References