Supply Chain Coordination Strategy under Demand Disruption—based on ABD Contracts

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Abstract

This paper considers the coordination problem of one-retailer one-supplier supply chains with stochastic demand and demand disruption. Adopted optimization methods and numerical experiments, we evaluate the benefits of the ABD contracts with and without demand disruption, and characterize the optimal discount price that maximizes the supplier’s expected profit under such circumstance. The results suggest that supply chain coordination can be achieved by advance booking discount contract under normal circumstances (without demand disruption) and under demand disruption situations. Unlike other coordination mechanisms, it does not need to devise an anti-disruption contract and change the parameters of the original contract, that is, the original ABD contract can coordinate the supply chain with or without demand disruption, so ABD contract is more robust and simple to implement. Further, by ABD contract, the supplier can share the deviation penalty costs with the retailer, and meanwhile acquire the total profits generated by at-once orders. At last, we give the conclusions and discuss the future direction.

Keywords: Supply Chain Coordination, ABD Contracts, Stochastic Demand, Demand Disruption

1. Introduction

Over the past few decades, entrepreneurs have paid a lot of efforts to make the supply chain lean and efficient. However, recent studies indicate that although those practices reduce operating costs successfully, they also result in increased supply chain vulnerability. Insurance company such as Munich Re report that, as natural disasters take place more frequently, the corresponding costs are also growing faster[1]; At the same time, man-made accidents (such as Military conflicts and terrorism attacks, etc.) also take place more frequently. Disruptive events within a supply chain can significantly impact the performance of the supply chain members (Chen and Xiao, 2009) [2]. So it is important to study the strategies to cope with supply chain disruption.

In this paper, the coordination contract of the supply chain with stochastic demand and demand disruption is considered. Demand disruption is caused by emergency events such as natural disasters and man-made accidents. Demand disruption can have two forms: demand rises or declines dramatically. A typical example of demand destruction is the milk scandal in China, the demand of domestic milk powder was declining sharply in 2008 due to the melamine-containing milk powder. Because there is deviation penalty cost resulting from deviation from original production plans, the coordination contract of supply chain with demand disruption is different from that without disruption. In this paper, although the deviation penalty cost is borne by the supplier, she can share it with the retailer and acquire the total profits generated by at-once order with advance booking discount (ABD) contract. Unlike other coordination mechanism, it does not need to devise an anti-disruption contract and change the parameters of the original contract, that is, the original ABD contract can also coordinate the supply chain with demand disruption, in this sense, ABD contract is more robust and simple to administer.
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2. Literature Review

This paper is closely related to supply chain coordination management and demand disruption management. The main purpose of supply chain management is to coordinate supply and demand, but in most cases, the supply chain can’t be fully coordinated due to double marginalization (Spengler, 1950) [3], that’s the reason why so many scholars investigate the supply chain coordination mechanism. Cachon (2003) makes a detailed research about supply chain contracts [4]. In most of supply chain coordination models, the supplier’s entire production is shipped to the retailer and the retailer never receives a second replenishment, and further, they all have additional administrative costs (such as handling, and monitoring costs) that are not explicitly considered. For that reason, Cachon (2004), Dong and Zhu (2007) use advance booking discount contract in supplier-retailer coordination [5], [6]. Originally, ABD contract is mainly used in retail industry, which means that the retailer entices customers to commit to their orders at a discounted price prior to the selling season, and these reserved orders are non-refundable and filled during the selling season(Tang et al., 2004) [7]. ABD contract can also coordinate the supplier-retailer supply chain through appropriate contract design. Cachon (2004) verifies ABD contract is simple to administer compared with other contracts [5].

All these contracts listed above are used to coordinate the supply chain in a static manner, that is, after the schedule has been settled down, the environment retains unchanged. For the fundamental supply chain, it can be coordinated through all above contracts. However, the environment is often disrupted by some unexpected events, such as strike, raw materials shortage, natural disasters like earthquakes and hurricanes. The disruptions have made companies aware of the need for active supply chain disruption management. Recently, some scholars discuss the supply chain coordination under demand disruption: Qi et al. (2004) introduce the idea of disruption management into supply chain coordination management [8]. Xiao and Qi (2008) studied the disruption management of the supply chain with two competing retailers, where the manufacturer faces a production cost disruption [9]. Chen and Xiao (2009) consider two kinds of coordination schedules: linear quantity discount schedule and Groves’s wholesale price schedule; under the linear quantity discount schedule, the manufacturer needs to adjust the maximum variable wholesale price after demand disruption [10].

Most literatures about demand disruption coordination contracts regard demand as deterministic and the demand disruption quantity is a random proportion of the normal demand quantity [8]-[10]; Further, they assume that the supplier’s entire production is shipped to the retailer and the retailer never receives a second replenishment, and the deviation cost is borne by the retailers rather than the supplier (Xiao et al., 2007) [11]. Our paper is different from other literatures studying demand disruption coordination in the following three aspects: Firstly, we consider stochastic demand and demand disruption, where demand disruption can change the distribution function of the original stochastic demand distribution, and the ABD contract gives the retailer a second-order chance (at-once order) which is not considered in the previous demand disruption contracts; Second, the supplier does not need to devise an anti-disruption contract and change the parameters of the original contract, that is, the original ABD contract can coordinate the supply chain with or without demand disruption, so ABD contract is more robust and simple to implement. Finally, we consider the penalty costs can be shared between the supplier and the retailer by ABD contracts instead of being borne by the supplier or the retailer only, and find that the supplier can acquire all the profits generated from the at-once order made by the retailer.

3. Models and Analysis

3.1. Symbols and Assumptions

Consider a single supplier and a single retailer supply chain that is susceptible to demand disruption. The retailer sells seasonal product, the supplier has only one production chance because of the long production time, the retailer must order from the supplier long before the selling season starts. However, due to the short replenishment time, the retailer has a second chance to replenish (we denote it as at-once order). We give our assumptions as follows:

1) The random demand D has a cumulative distribution function (c.d.f) of F(x) and the probability
density function (p.d.f) is f(x); F(x) is strictly increasing and second differentiable, and F(0)=0.
(2) The supplier and the retailer are both risk-neutral, and as long as \( \pi_{i} \geq 0 (i=s,r) \), they all prefer to join in the market transaction.
(3) The information is symmetric among members in the supply chain, that’s to say, the costs, structure, demand information and profit function about both partners in the supply chain are common knowledge.

The variables used in the paper are summarized in Table 1.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( q_{1s} ) / ( q_{2s} ) / ( q_s )</td>
<td>Under ABD contract without demand disruption, the pre-booking quantity / the at-once order quantity / the total quantity the supplier willing to provide. ( q_s = q_{1s} + q_{2s} )</td>
</tr>
<tr>
<td>( q_{1sd} ) / ( q_{2sd} ) / ( q_{sd} )</td>
<td>Under ABD contract with demand disruption, the pre-booking quantity / the at-once order quantity / the total quantity the supplier willing to provide. ( q_{sd} = q_{1sd} + q_{2sd} )</td>
</tr>
<tr>
<td>( S(q) )</td>
<td>The expected sale quantity</td>
</tr>
<tr>
<td>( c )</td>
<td>Production cost per unit of the supplier</td>
</tr>
<tr>
<td>( p )</td>
<td>Regular selling price per unit of the retailer</td>
</tr>
<tr>
<td>( v )</td>
<td>Salvage value per unit of the product</td>
</tr>
<tr>
<td>( w_{1} ) / ( w_{2} )</td>
<td>the wholesale price the supplier provides to the retailer with advance booking / at-once orders, ( p \geq w_{1} &gt; w_{2} ) &gt; ( c &gt; v )</td>
</tr>
<tr>
<td>( q_{1r} ) / ( q_{2r} ) / ( q_{r} )</td>
<td>Under ABD contract without demand disruption, the advance booking quantity / the at-once ordering quantity / the total quantity the retailer willing to order. ( q_{r} = q_{1r} + q_{2r} )</td>
</tr>
<tr>
<td>( q_{1rd} ) / ( q_{2rd} ) / ( q_{rd} )</td>
<td>Under ABD contract with demand disruption, the advance booking quantity / the at-once ordering quantity / the total quantity the retailer willing to order. ( q_{rd} = q_{1rd} + q_{2rd} )</td>
</tr>
<tr>
<td>( a_{1} ) / ( a_{2} )</td>
<td>Under demand disruption, the extra costs incurred by the supplier when demand rises / declines sharply</td>
</tr>
</tbody>
</table>

The sequence of events is assumed to be as follows: firstly, the supplier provides the ABD contract \( \{ w_{1}, w_{2} \} \) to the retailer, where \( w_{1} / w_{2} \) is the wholesale price the supplier provides to the retailer with advance booking / at-once orders respectively. Observed \( w_{1} / w_{2} \), the retailer decides the advance order batches \( q_{1} \) (the price is \( w_{1} \) in this period), the supplier starts to produce and delivers it to the retailer before selling season starts. At the beginning of the selling period, the demand of the products is satisfied first from the retailer’s stock, having observed the true market demand and the supplier’s supply states, the retailer can send an at-once order to the supplier if the supplier has some inventory. Per unit wholesale price at this time is \( w_{2} \). If the supplier has stock, the order can be filled at once, and does not incur additional handling costs relative to the pre-book order, if the supplier is out of stock, then the demand is lost.

3.2. Baseline Case: the Supply Chain Coordination without Demand Disruption

3.2.1. The Centralized Decision-making Supply Chain Model

Consider the centralized decision-making supply chain whose objective function is to maximize the whole profits of the supply chain, and the decision variable is the ordering quantity \( q \).

The objective function of the entire supply chain is:

\[
\Pi (q) = (p - v) S(q) - (c - v)q
\]

Where,

\[
S(q) = q - \int_{0}^{q} F(x) dx
\]

(1) is a simple newsvendor model, \( \Pi (q) \) is a concave function of \( q \), according to the first order condition (F.O.C), the first-best ordering quantity of the whole supply chain is
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\[ F(q^*) = \frac{p - c}{p - v} \cdot q^* = F^{-1}\left(\frac{p - c}{p - v}\right) \tag{3} \]

Right now, the maximum expected profit of the supply chain is \( \Pi^0 \), which satisfies:
\[ \Pi^0(q^*) = (p - v)S(q^*) - (c - v)q^* \tag{4} \]

3.2.2. ABD Contract to Coordinate the Decentralized Decision-making Supply Chain

When there is no coordination contract, the decentralized decision-making supply chain can’t be coordinated because of double marginalization (Spengler, 1950) [3]. At this time, the retailer’s optimum ordering quantity is
\[ \tilde{q}_r = F^{-1}\left(\frac{p - w}{p - v}\right) \tag{5} \]

\( \tilde{q}_r \) is less than the optimal order quantity \( q^* \) \((w>c)\). The supply chain can’t obtain the optimal profits as in centralized decision-making supply chain. Next, we will show the supply chain can be coordinated by ABD contract.

The supplier provides an ABD contract \([w_1, w_2]\) \((w_1<w_2)\) to the retailer, under this contract, the retailer will place an irrevocable order prior to the supplier starts to produce, and the committed amount is \(q_{r1}\). The supplier will produce \( q_s \) no less than \( q_{r1} \) to satisfy the demand, and by setting appropriate wholesale price \( w_1 \), the supplier can let the pre-booking quantity made by the retailer is less than the expected sales. Hence, we directly use \( q_{r1} > q^* \). The supplier’s profit function is:
\[ \pi_s(q_{r1}, q_{r2}) = (w_1 - v)q_{r1} + (w_2 - v)(S(q_r) - S(q_{r1})) - (c - v)q_r \tag{6} \]

By simple calculation, the Hessian matrix of \( \pi_s(q_{r1}, q_{r2}) \) is negative defined for \( q_{r1} \) and \( q_{r2} \). According to the F.O.C the optimal supply batch is:
\[ q_{s1} = F^{-1}\left(\frac{w_2 - w_1}{w_2 - v}\right) \quad q_{s2} = F^{-1}\left(\frac{w_2 - c}{w_2 - v}\right) \tag{7} \]

From (7), the optimal supply quantity about the supplier \( q_s \) is irrelevant of the advance booking discount wholesale price \( w_1 \).

The profit function of the retailer is:
\[ \pi_r(q_{r1}, q_{r2}) = (p - v)S(q_{r1}) + (p - w_2)(S(q_r) - S(q_{r1})) - (w_1 - v)q_{r1} \tag{8} \]

Similarly, the Hessian matrix of \( \pi_r(q_{r1}, q_{r2}) \) is negative defined about \( q_{r1} \) and \( q_{r2} \). According to the F.O.C, the optimal advance booking batch is:
\[ q_{r1} = F^{-1}\left(\frac{w_2 - w_1}{w_2 - v}\right) \tag{9} \]

After observed the real demand information and the production quantity of the supplier, the retailer’s at-once order quantity satisfies: \( q_{r2} = \min( x - q_{r1}^*, q_{r2}^*) \), where
\[ q_{2s}^* = q_s^* - q_{1s}^* \]  

(10)

Compare (3) with (7), we can conclude that, when the supplier sets \( w = P \), she will produce the optimal quantity equaling to the quantity of the centralized supply chain, and further, the optimal advance order quantity exactly equals to the supply quantity of the supplier. The expected profit function of the supplier is:

\[ \pi^*_s (q_{1s}^*, q_s^*) = (w_1 - v)q_{1s}^* + (p - v)(S(q_s^*) - S(q_{1s}^*)) - (c - v)q_s^* \]  

(11)

The expected profit function of the retailer is:

\[ \pi^*_r (q_{1s}, q_s) = (p - v)S(q_{1s}^*) - (w_1 - v)q_{1r}^* \]  

(12)

The expected profit function of the whole supply chain is:

\[ \Pi^*_w = \pi^*_s + \pi^*_r = (w_1 - v)q_{1s}^* + (p - v)(S(q_s^*) - S(q_{1s}^*)) - (c - v)q_s^* + (p - v)S(q_{1s}^*) - (w_1 - v)q_{1r}^* \]  

(13)

\[ \therefore q_{1r}^* = q_{1s}^*, \therefore \Pi^*_w = (p - v)S(q_{1s}^*) - (c - v)q_s^* \]  

(14)

We have the following proposition.

Proposition1: When the supplier provides ABD contract to the retailer and sets the wholesale price of the at-once order as \( w = P \), the supply chain can be coordinated, and by means of setting the advance booking discount price, the supplier can split profit arbitrarily among herself and the retailer.

Proof: Compare (3) with (7), the supply quantity the supplier willing to provide exactly equals to the optimal ordering quantity \( 0q \) of the centralized decision making supply chain. From (9), it can be seen that, the advance booking quantity by the retailer \( q_{1r}^* \) just equals to the optimal ordering quantity \( q^* \) without ABD contract \( \bar{q} \). However, after observed the precise market demand and the total supply batch provided by the supplier, the retailer has a second replenishment opportunity, the maximum order quantity can amount to the optimal order quantity of the centralized decision-making supply chain.

Next we will prove the total profits of the supply chain under ABD contract equals to those of the centralized decision-making supply chain.

From (14), the total profits of the supply chain under ABD contract is:

\[ \Pi^*_w = (p - v)S(q_{1s}^*) - (c - v)q_s^* \]

\[ \therefore q_{1s}^* = q^* \]  

and  

\[ \Pi^*_w = \pi^*_s (q^*) = (p - v)S(q^*) - (c - v)q^* \]

From (7) and (9), we can conclude that the advance booking quantity of the retailer equals to the supply quantity the supplier willing to provide, namely, \( q_{1s}^* = q_{1r}^* = F \left( \frac{w_1 - w}{p - v} \right) \). Supplier can split the supply chain profits arbitrarily between himself and the retailer through setting suitable \( w \), that’s to say, ABD contract can coordinate the supply chain.

Lemma 1: The supplier can acquire the entire profits generated from at-once order quantity made by the retailer under ABD contract, in other words, the retailer can only get zero profit from the at-once order opportunity.

We can get lemma 1 from (8) and (12) easily, when the supplier offers ABD contract to the retailer, she just needs to set \( w_1 = p \), the retailer will accept the contract, and the supplier can acquire the entire profit from the at-once order.

3.3. The Supply Chain Coordination with ABD Contract with Demand Disruption

Assume the supplier has arranged production plan in terms both of the forecasting market demand and the pre-booking quantity ordered by the retailer, her planning production is \( \bar{q} \). Unexpectedly, some disruptive events occur, the demand can rise or decline dramatically, which change the cumulative
probability function $F$. Assume the new demand c.d.f and p.d.f is $G(y)$ and $g(y)$ respectively. Similarly, $G(y)$ is a strictly increasing and twice differentiable function, and $G(0) = 0$. When market demand rises dramatically, we have $G(y) < F(y)$ (Order fill rate decreases); If there is a deep drop in demand, we have $F(y) < G(y)$ (Order fill rate increases) for arbitrary $y$.

3.3.1. The Centralized Decision-making Supply Chain with Demand Disruption

The decision variable of the centralized decision-making supply chain is the ordering quantity $q_d$.

The objective function of the supply chain is:

$$
\Pi^d(q) = (p - v)S_q(q) - (c - v)q - a_1(q - q^0)^+ - a_2(q^0 - q)^+
$$

(15)

$q_d^* = q^0$, where $q^0$ denote the optimal ordering quantity of the whole supply chain without demand disruption. Generally, the system disruption will result in the deviation penalty costs due to the production quantity change. We assume $\epsilon^0$ or $\epsilon^0$ are the unit extra cost undertaken by the supplier owing to the demand disruption (rise or decline dramatically). That is if demand is rising to $q^* > q^0$, the supplier needs to utilize the back-up production capacity to fill this rising demand, the unit cost of the back-up production is $\epsilon^0$; if demand decrease to $q^* < q^0$, the unit extra cost is $\epsilon^0$, resulting from the supplier has to sell the surplus in a second market or depreciate it as salvage, etc. Assume $\epsilon^0, \epsilon^1 > 0$, and let $\epsilon^* = \max(\epsilon^0, \epsilon^1)$.

Lemma 2: Assume the optimal ordering quantity under demand disruption is $q^\ddagger$, if demand increases suddenly due to disruptive events, that is $G(q) < F(q)$, we have $q^\ddagger > q^0$; if demand decreases suddenly due to disruptive events, that is $F(q) < G(q)$, we have $q^\ddagger < q^0$, which has already been proved by Yu et al (2005) [12].

Proposition 2: The optimal ordering quantity in the centralized decision-making supply chain is:

$$
q_d = \begin{cases} 
\hat{q}^d, & \hat{q}^d > q^0 \\
\tilde{q}^d, & \tilde{q}^d < q^0 \\
q^0, & \text{else}
\end{cases}
$$

(16)

Where

$$
\hat{q}^d = G^{-1}\left(\frac{p - c + a_2}{p - v}\right); \quad \tilde{q}^d = F^{-1}\left(\frac{p - c + a_2}{p - v}\right)
$$

Proof: Similarly, $F(q)$ is concave in $q$, according to the F.O.C, we have:

(1) From lemma 2, when $G(q) < F(q)$, we have $q^\ddagger > q^0$, that is, when demand increases suddenly ($\epsilon^0 > \epsilon^0$), the optimal order quantity of the supply chain is:

$$
G(\hat{q}^d) = \frac{p - c + a_2}{p - v}; \quad \hat{q}^d = G^{-1}\left(\frac{p - c + a_2}{p - v}\right)
$$

(17)

(2) When $F(q) < G(q)$, we have $q^\ddagger < q^0$, that is, when demand decreases suddenly ($\epsilon^0 > \epsilon^0$), the optimal order quantity of the supply chain is:

$$
G(\tilde{q}^d) = \frac{p - c + a_2}{p - v}; \quad \tilde{q}^d = G^{-1}\left(\frac{p - c + a_2}{p - v}\right)
$$

(18)

(3) When the severity of demand disruption is nothing serious, that is $\epsilon^0 > \epsilon^0$ and $\epsilon^0 > \epsilon^0$, the optimal order quantity of the supply chain is $q^0$, and the whole supply chain profit function is:
3.3.2. The Model of ABD Contract to Coordinate the Supply Chain with Demand Disruption

Here, the supplier still uses ABD contract to coordinate the supply chain, the profit function of the supplier is:

\[
\pi_{sd}(q_{1sd}, q_{sd}) = (w_1 - v)q_{1sd} + (w_2 - v)(S_{G}(q_{sd}) - S_{G}(q_{1sd})) - (c - v)q_{sd} - a_1(q_{sd} - q^*_1) - a_2(q^*_2 - q_{sd}^*)
\]

(20)

When demand increases suddenly \((q_{sd} > q^*_2)\), it's easy to know the Hessian Matrix of \(\pi_{sd}(q_{1sd}, q_{sd})\) with \(q_{sd} > q_{sd}^*\) is negative definite. According to the F.O.C, the advance supplying quantity of the supplier is:

\[
q_{1sd}^* = G^{-1}\left(\frac{w_2 - w_1}{w_2 - v}\right)
\]

(21)

The total supply quantity of the supplier is:

\[
q_{sd}^* = G^{-1}\left(\frac{w_2 - w_1}{w_2 - v}\right)
\]

(22)

When demand increases suddenly \((q_{sd} < q^*_1)\), similar with (18), the advance supplying quantity of the supplier is:

\[
q_{1sd}^* = G^{-1}\left(\frac{w_2 - w_1}{w_2 - v}\right)
\]

(23)

The total supply quantity of the supplier is:

\[
q_{sd}^* = G^{-1}\left(\frac{w_2 - w_1 + a_2}{w_2 - v}\right)
\]

(24)

The profit function of the retailer with demand disruption is:

\[
\pi_{rd}(q_{1rd}, q_{rd}) = (p - v)S_{G}(q_{rd}) + (p - w_2)(S_{G}(q_{rd}) - S_{G}(q_{1rd})) - (w_1 - v)q_{1rd}
\]

(25)

It's easy to see that, \(\pi_{rd}(y, q)\) is concave in \(q_{1rd}\). The optimal pre-booking quantity of the retailer is:

\[
q_{1rd}^* = G^{-1}\left(\frac{w_2 - w_1}{w_2 - v}\right)
\]

(26)

Hence, at-once order quantity of the retailer when she has observed the actual demand and the supply quantity of the supplier is:

\[
q_{2rd} = \min(\ X_{G} - q_{1rd}^*, q_{2sd}^* )\ ,\text{ where, }\ q_{2sd}^* = q_{sd}^* - q_{1sd}^*
\]

(27)

By comparison (16) with (27), we can see that, the supplier just needs to maintain the wholesale price \(w_2\) to the retail price \(p\), the supplier will provide the overall supply chain optimal quantity, and
the pre-booking quantity by he retailer exactly equals to what the supplier willing to supply.

Proposition 3: The decentralized decision-making supply chain under ABD contract with demand disruption, the pre-booking quantity and the total quantity the supplier willing to provide are:

\( q_{sd}^* = G^{-1}(w_2 - w_r - \frac{w_2 - c - a_1}{w_2 - v}) \), when demand decreases quickly, we have \( q_{sd}^* > q_{td}^* \); when demand increases quickly, we have \( q_{sd}^* < q_{td}^* \); when the disruption is not severe, we have \( q_{sd}^* = q_{td}^* \).

\( q_{sd}^* = G^{-1}(w_2 - c + a_2) \), when demand decreases quickly, we have \( q_{sd}^* > q_{td}^* \); when demand increases quickly, we have \( q_{sd}^* < q_{td}^* \); when the disruption is not severe, we have \( q_{sd}^* = q_{td}^* \).

Proof: The proof of (2) is similar to proposition 1, so omitted here; next we will give the proof of (1)

From (7), when \( G(q) < F(q) \) (demand increases quickly),

\( q_{sd}^* = G^{-1}(w_2 - c - a_1) \), when demand decreases quickly, we have \( q_{sd}^* > q_{td}^* \); when the disruption is not severe, we have \( q_{sd}^* = q_{td}^* \).

Proposition 3 indicates that, when demand is disrupted, the quantity the supplier willing to provide is changing accordingly. Interestingly, when demand increases/decreases suddenly, the advance booking quantity the supplier is willing to provide decreasing/increasing on the contrary; however, the total supply quantity is increasing or decreasing accordingly. A possible explanation is, when demand increases/decreases suddenly, the supplier can obtain higher profits through decreasing/increasing the at-once order quantity (Lemma 1).

Now, the expected profit function of the supplier is:

\[ \pi^s(q_{sd}^*, q_{td}^*) = (w_1 - v)q_{td}^* + (p - v)S(c(q_{sd}^*)) - S(c(q_{td}^*)) - (c - v)q_{sd}^* - a_1(q_{sd}^* - q_{td}^*) - a_2(q_{td}^* - q_{td}^*) \] (28)

The expected profit function of the retailer is:

\[ \pi^r(q_{td}^*, q_{td}^*) = (p - v)S(c(q_{td}^*)) - (w_1 - v)q_{td}^* \] (29)

The expected profit of the whole supply chain is:

\[ \Pi^w = \pi^s + \pi^r = (w_1 - v)q_{td}^* + (p - v)(S(c(q_{sd}^*)) - S(c(q_{td}^*)) - (c - v)q_{sd}^* - a_1(q_{sd}^* - q_{td}^*) - a_2(q_{td}^* - q_{td}^*) + (p - v)S(c(q_{td}^*)) - (w_1 - v)q_{td}^* \] (30)

\[ \therefore \Pi^w = (p - v)S(c(q_{sd}^*)) - (c - v)q_{sd}^* - a_1(q_{sd}^* - q_{td}^*) - a_2(q_{td}^* - q_{td}^*) \] (31)

Consequently, we have proposition 4:

Proposition 4: When the supplier offers ABD contract to the retailer, and set \( w_2 = p \), the supply chain with demand disruption can be coordinated by ABD contract, and the supplier can split profits arbitrarily between herself and the retailer via setting proper \( w_1 \).

Proof: Compare (19) with (31), we can see, when \( w_2 = p \), the quantity the supplier willing to provide just equals to the optimal ordering quantity of the whole supply chain \( q^* \). From (26), the pre-booking quantity \( q_{td}^* \) of the retailer exactly equals to the optimal order quantity without advance booking. However, the retailer has a second chance to replenish after observed the actual market demand information and the overall supply quantity provided by the supplier, that’s to say, the maximum order quantity can amount to the optimal order quantity in the centralized decision-making supply chain.

Next, we will prove the profits of decentralized decision-making supply chain under ABD contract are equal to those obtained from centralized decision-making supply chain.

From (19), the overall supply chain profits under ABD contract is:
\[ \pi^*(q^*) = (p - v) \delta(q^*) - (c - v)q^* - a_1 q^* - a_2 q^* \]

And, \( q_{1*} = q^* \cdot \). From (31), \( \pi_{1*} = (p - v) \delta_{1*}(q_{1*}) - (c - v)q_{1*} - a_1(q_{1*} - q^*) - a_2(q_{1*} - q^*) \).

From (23) and (26), the pre-booking quantity ordered by the retailer equals to the pre-booking quantity the supplier willing to provide, that is, \( q_{1*} = q_{1*}^* = G^{-1}(\frac{p - w_1}{p - v}) \). The supplier can split profits arbitrarily between herself and the retailer through setting proper \( w_1 \), in other words, the supply chain can be coordinated by ABD contract.

Lemma 4: Under ABD contract with demand disruption, the supplier can acquire the entire profits generated from at-once order made by the retailer, in the other words, the retailer only gets zero profit from the at-once order opportunity.

The same as lemma 1, when demand is disrupted, the retailer only gets zero profit from the at-once order opportunity. However, by setting proper \( w_1 \), the supplier can share the costs caused by demand disruption with the retailer.

Therefore, the supply chain can be coordinated by ABD contract with demand disruption, and compared with other contracts (such as buyback contract, revenue sharing contract, etc.), the supplier does not need to devise an anti-disruption contract and change the parameters of the original contract, that is, the original ABD contract can coordinate the supply chain. For this reason, an ABD contract is more robust and simple to administrate.

4. Numerical Experiments

Assume a seasonal product (such as the Sports souvenirs provided by a large international sports meeting), demand for such products has great uncertainty. Assume demand is Normal Distribution \( N \sim (1000, 100^2) \) without demand disruption. In these experiments, we just consider demand decreases suddenly due to emergency events (such as before the sports meeting starts, a natural disasters like earthquakes take place and causes the sports meeting canceled, which will result in a sharp drop in demand for the Sports souvenirs). After demand disrupted, the distribution function is \( N \sim (600, 150^2) \), other parameters are given in table 2. Next, to simplify expression, we will use CD to denote centralize decision-making supply chain; DD to denote decentralized decision-making supply chain without ABD contract; DD-ABD to denote decentralized decision-making supply chain with ABD contract.

<table>
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<th>Table 2. Parameters Table</th>
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<td>( c )</td>
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<td>8</td>
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<tr>
<th>Table 3. The Parameters Comparison Before and After Demand Disruption (( w_1=10/w_1=15 ))</th>
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<tr>
<td>Without Demand Disruption</td>
</tr>
<tr>
<td>CD</td>
</tr>
<tr>
<td>( q_{1*}/q_{1*} )</td>
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<td>( q_{1*}/q_{1*} )</td>
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<td>( \pi_{1*}/\pi_{1*} )</td>
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From Table 3, we can conclude that, when demand falls suddenly, the profits of the retailer, the

\* The value in the brackets denote the objective function values for \( w_1=15 \).
supplier and the whole supply chain will decline accordingly. Whether demand disruption occurs or not, ABD contract can coordinate the decentralized supply chain. Through setting proper $w_1$, the supplier can split profits arbitrarily between herself and the retailer. To illustrate this point, we set $w_1=10$ and $w_1=15$ respectively, the numerical experiments results are given in Table 3.

Next, we set $w_1=[5, 18]$, and show how the profits of supplier, retailer, and the whole supply changes accordingly to $w_1$, we show the results in Figure 1. From Figure 1, with ABD contract, the profits of the retailer are falling in, on the contrary, the profits of the supplier are rising in $w_1$, and the whole supply chain profit remains unchanged. The reason is the benefits generated from advance booking of the retailer is decreasing in $w_1$, the supplier is on the contrary. It is further confirmed that the supplier can split profits arbitrarily between herself and the retailer via setting proper.

![Figure 1. The Profits of the Supplier, Retailer, the Whole Supply Chain with Demand Disruption](image)

5. Conclusions

In this paper, we consider the coordination problem of one-retailer one-supplier supply chain with stochastic demand and demand disruption. Demand variation often makes the real production quantity different from what is originally planned, causing a deviation cost on the supplier. We evaluate the benefits of the ABD contracts with and without demand disruption, and characterize the optimal discount price that maximizes the supplier’s expected profit under such circumstance. Supply chain coordination can be achieved by advance booking discount contract under normal circumstances (without demand disruption) and under demand disruption situations. Further, with ABD contract, the profits of the retailer are declining in $w_1$, on the contrary, the profits of the supplier are rising in $w_1$ and the supplier can acquire the entire profits generated from at-once order made by the retailer. Unlike other coordination mechanisms, the supplier does not need to devise an anti-disruption contract and change the parameters of the original contract, that is, the original ABD contract can coordinate the supply chain, so ABD contract is more robust and simple to implement.

Our research in this paper can be extended at least in the following aspect: If the retailer can set the retail price, the supplier can’t extract all the profits generated from the at-once order quantity by the retailer with or without demand disruption. In this case, whether the ABD contract can coordinate the supply chain or not needs further investigation.

6. References


