Dynamic Pricing for Perishable Products with Cancelations and No-shows

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Abstract
Assume that customers are price sensitive, changes in the price of perishable products will bring changes in demand, and business can adjust the perishable products' price to influence the market demand, a perishable products multi-stage pricing model considering cancelations and no-shows is proposed, and the optimal pricing, the optimal expected revenue with the products' initial quantity are analyzed in this paper.

Keywords: Revenue Management, Dynamic Pricing, Perishable Products

1. Introduction
Revenue management is the application of information systems and pricing strategies to allocate the right capacity to the right customer at the right price at the right time [13]. An overview of revenue management is presented by Boyd and Bilegan [4].

Pricing becomes more and more important for perishable products especially dynamic pricing. The problem of optimal dynamic pricing is very intellectual, economic, and practical. Dynamic pricing comes from difference pricing. In the last decades, optimal dynamic pricing remains a challenging problem. A survey of this research is given in Bitran and Caldentey [3]). They give an elaborate overview of revenue management and examine the research and results of dynamic pricing policies and their relation to revenue management in their paper. And the book by Talluri and van Ryzin [17] proposed comprehensive overviews of the areas of dynamic pricing and revenue management also. Feng and Xiao [8] study pricing problems with a predetermined set of price points. Xiao and Yang [19] provide a continuous-time stochastic control model to study the revenue management problem with two capacity features, show that if the revenue rate is concave in the capacity usage, the expected value of marginal capacity is monotone.

Feng and Gallego [9] use a diffusion model to characterize the intensity of the demand process. Feng and Gallego [11] study a single-product, two-price model where the prices in both periods are fixed and the only decision is when to switch from one to the other. Feng and Xiao [7] extend the model presented in Feng and Gallego [11] to the case of a risk-sensitive seller who penalizes the variance of revenue linearly. Schon [17] present a single-resource finite-horizon Markov decision process approach for a firm that seeks to maximize expected revenues by dynamically adjusting the menu of offered products and their prices that are selected from a finite set of alternative values predetermined as a matter of policy assuming that consumers choose among available products according to an attraction choice model. Heo and Lee [12] conduct a logit analysis by comparing two groups to explore which consumer characteristics influence fairness perceptions of revenue management pricing in hotel context. Wang et al. [19] propose a theoretical model to research selecting and using behavior considering the tariff choice behavior under nonlinear pricing frame discuss under non-linear pricing frame.

An arriving customer at time \( t \) has a reservation price \( p_t \) for the product, i.e., the maximum price the customer is willing to pay. Generally assume that the seller know that the reservation price \( p_t \) is a random variable with distribution \( F(t) \), and the distribution is often assumed to be Poisson process.
These arguments can be found in Kincaid and Darlings [18], Gallego and van Ryzin [10], and Bitran and Mondschein [2]. Bitran and Mondschein [2] consider a periodic pricing review policy where prices are revised only at a finite set of decision times. Similar to Kincaid and Darling [18], the demand model is the combination of a Poisson arrival process of customers and a purchasing process based on a reservation price, which is unknown to the seller. Recently, Feng and Xiao [9] give a comprehensive model to integrate pricing and capacity allocation for perishable products.

Much of the lectures assume that the seller is risk neutral except Feng and Xiao [7] and Lai and Ng [15]. Lai and Ng propose a stochastic approach and use an absolute deviation model to measure risk revenue from random demands in several scenarios. Li [16] provides a cruise line overbooking risk decision model with multiple price classes, incremental cost and nonlinear goodwill loss by employing the real options approach to reduce and avoid the risks of cruise line overbooking.

In terms of business practice, varying pricing is often the most natural mechanism for revenue management. Lots of retail and industrial firms use various forms of dynamic pricing to respond to market fluctuations or demand uncertainty. Dynamic pricing strategies are really useful to balance utilization and profitability of the limited capacity to control the uncertainty demand by adjusting pricing of the available capacity. In the last decade pricing polices became an important component of revenue management. But the retailer or industries can not usually vary their prices, because frequent alterations lead to higher search costs for customers, and dissatisfy customers so as to destroy the firms’ reputation. So in our model, we assume that the price in each period is fixed and varies from period to period.

In this research, we study the optimal pricing strategy for ordering perishable products using dynamic programming. During every reservation period, when a potential customer requests a product, the manager has to decide which price should be made and whether or not rent it to that potential customer. When making this decision, the manager does not know how many additional potential customers will arrive that day or during the coming days. If the perishable product is sold to a low payoff customer, a high payoff customer may be turned away if there is a shortage of products. If the product is not sold at that time, the low payoff customer will be turned away. The major contribution of this paper is that the model of dynamic pricing formulated in this paper considering customers’ no-shows and cancelations.

This paper is organized as follows. In the following section, we propose some related notations, assumptions and preliminary results. An example is illustrated in section 3 and the conclusions are presented in section 4.

2. Assumptions, formulations and preliminary results

In our model, the finite reservation horizon for perishable products is divided into $T$ periods. At the beginning of each period the unit selling price is decided, at the same time, the quantity of reservation products is made. We assume that the demand in every period is independent, identically distributed, and are sensitive to the price set for the corresponding period. Namely, the intensity of demand varies by different price in each period. The function $V_t(c)$ is defined as the maximum expected revenue if the hotel starts with $c$ products at the beginning of period $t$. We assume that price can be changed from period to period, so the problem can be formulated as a Markov decision problem (MDP) with $c_j$ as the state of the system at the beginning of $t$, where $c_j$ is the number of remaining products at period $t$.

The major notations for parameters and variables used in this study are as follows:

- $C$ number of perishable products available at time $T$;
- $T$ number of reservation periods;
- $q_t$ proportion of no-shows or cancelations of period $t$ at the end of reservation horizon;
- $p_t$ reservation price to each product at period $t$;
- $p_{\text{min}}$ lowest possible unit price to be charged;
- $p_{\text{max}}$ highest possible unit price to be charged;
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(iv) \( \frac{\partial}{\partial p} G(c, p) \) is increasing in \( c \) and decreasing in \( p \).

Proof: (i) From (2)

\[ v(c_i, p_i) = E[p_i \min\{d(p_i), c_i\} - p_iq_i \min\{d(p_i), c_i\}] \]

= \( (1 - \alpha q_i)E[p_i \min\{d(p_i), c_i\}] \).

By assumption 2, \( p_i d(p_i) \) is concave in \( p_i \). \( p_i c_i \) is jointly concave in \((c_i, p_i)\) because the Hessian is negative semi-definite. Therefore \( \min\{p_i d(p_i), p_i c_i\} \) is jointly concave because it is the minimum of two concave functions. Hence, \( E[\min\{p_i d(p_i), p_i c_i\}] \) is jointly concave in \((c_i, p_i)\). Therefore \( v(c_i, p_i) \) is jointly concave in \((c_i, p_i)\).

(ii) Define \( \hat{c}_i = \lambda c_i + (1 - \lambda)c^* \), \( \hat{p}_i = \lambda p_i + (1 - \lambda)p^* \), where \( 0 \leq \lambda \leq 1 \). Then

\[ G_{t+i}(\hat{c}, \hat{p}) = E[V_{t+i}(\hat{c} - (1 - q_i) \min\{d(\hat{p}), \hat{c}_i\})] \]

\[ \geq E[V_{t+i}(\hat{c} - (1 - q_i) \min\{\lambda d(p_i) + (1 - \lambda)d(p^*), \hat{c}_i\})] \]

= \( E[V_{t+i}(\lambda c_i - (1 - q_i) \min\{d(p_i), c_i\}) + (1 - \lambda)(c^2 - (1 - q_i) \min\{d(p_i), c_i\})] \]

\[ \geq \lambda E[V_{t+i}(c_i - (1 - q_i) \min\{d(p_i), c_i\}) + (1 - \lambda)V_{t+i}(c^2 - (1 - q_i) \min\{d(p_i), c_i\})] \]

= \( \lambda G_{t+i}(c^i, p_i) + (1 - \lambda)G_{t+i}(c^2, p^*) \),

where the first inequality follows from assumption 1 that \( d(p) \) is convex in \( p \in [p_{\min}, p_{\max}] \) and \( \alpha q_i \leq 1 \), and the second inequality follows from (iii) which is proved next. Therefore \( G_{t+i}(c, p) \) is jointly concave in \((c, p)\).

(iii) Consider any two nonnegative values \( c^i \) and \( c^j \), let \( p^{*i} \), \( i = 1, 2 \) denote the optimal solution of

\[ \max_{p \in [p_{\min}, p_{\max}]} E[v(c_i, p) + V_{t+i}(c_i - (1 - q_i) \min\{d(p), c_i\})], \quad i = 1, 2. \]

Define \( \hat{c}_i = \lambda c_i + (1 - \lambda)c^* \), \( \hat{p}_i = \lambda p_i + (1 - \lambda)p^{*i} \), where \( 0 \leq \lambda \leq 1 \). Then

\[ V_{t+i}(\hat{c}) = \max_{p \in [p_{\min}, p_{\max}]} E[v(\hat{c}, p) + V_{t+i}(\hat{c} - (1 - q_i) \min\{d(p), \hat{c}_i\})] \]

\[ \geq E[v(\hat{c}, \hat{p}) + V_{t+i}(\hat{c} - (1 - q_i) \min\{d(\hat{p}), \hat{c}_i\})] \]

\[ \geq \lambda E[v(c^1, p^{*1}) + V_{t+i}(c^1 - (1 - q_i) \min\{d(p^{*1}), c^1\})] \]

+ \( (1 - \lambda)E[v(c^2, p^{*2}) + V_{t+i}(c^2 - (1 - q_i) \min\{d(p^{*2}), c^2\})] \]

= \( \lambda V_{t+i}(c^1) + (1 - \lambda)V_{t+i}(c^2) \),

where the last equality follows from (i) and (ii). Therefore \( V_t(c) \) is concave in \( c \).

(iv) From (ii) we know that \( G_{t+i}(c, p) \) is jointly concave in \((c, p)\), therefore \( \frac{\partial}{\partial p} G(c, p) \) is decreasing in \( p \).
From assumption 1, \( d(p) \) is decreasing in \( p \), so \( d(p + \alpha) \leq d(p) \), therefore
\[
c - (1 - p_r) \min\{d(p + \alpha), c\} \geq c - (1 - p_r) \min\{d(p), c\}.
\]

From (ii) and (iii), we obtain that the difference
\[
V_{t+1}(c - (1 - q_t) \min\{d(p + \alpha), c\}) - V_{t+1}(c - (1 - q_t) \min\{d(p), c\})
\]
is increasing in \( c \).
Therefore, taking expectations above we have that the difference
\[
G_{t+1}(c, p + \alpha) - G_{t+1}(c, p)
\]
is increasing in \( c \).
And
\[
\frac{\partial}{\partial p} G(c, p) = \lim_{\alpha \to 0} \frac{1}{\alpha} (G_{t+1}(c, p + \alpha) - G_{t+1}(c, p)),
\]
therefore, \( \frac{\partial}{\partial p} G(c, p) \) is increasing in \( c \).

The algorithm of above dynamic programming model is as follows:
Let \( V_{T+1}(c) = 0, \ c = 0, 1, \ldots, C \),

**Step1.** Calculate \( V_T(c) \) and \( p^*_T(c) \), \( k + 1 := T \),

**Step2.** Calculate \( V_k(c) \) and \( p^*_k(c) \), where
\[
V_k(c) = \max_{\mu \in [\mu^*, \mu^*]} E[V(c, p) + V_{k+1}(c - (1 - q_t) \min\{d(p), c\})],
\]

**Step3.** If \( k \leq 1 \), then end; else, \( k := k - 1 \), turn to step 2.

3. Numerical examples

In this section we present numerical examples to illustrate the dynamic pricing model proposed in our paper. Assume that \( \alpha = 0.8 \), \( q_t = 0.1, t = 1, 2 \). For the above problem, we just consider a 2-period reservation horizon, where during the first period \( \mu_1 \) is given by \( \mu_1 = 10 - p_1 \), and in period 2, \( \mu_2 \) is given by \( \mu_2 = 12 - 2p_2 \).

From model (4),
\[
V_1(c) = 0, \ c = 0, 1, \ldots, C,
\]
\[ V_2(c) = \max_{p \in [p^{min}, p^{max}]} E[v(c, p) + V_3(c - (1 - q_z) \min\{d(p), c\})] \]
\[ = \max_{p \in [p^{min}, p^{max}]} E[v(c, p)] \]
\[ = \max_{p \in [p^{min}, p^{max}]} (1 - \alpha q_z) E[p \min\{d(p), c\}] \]
\[ = \max_{p \in [p^{min}, p^{max}]} (1 - \alpha q_z) p \left[ c + \sum_{k=0}^{c} (k - c) \left( \frac{(\mu_z(p))^k e^{-\mu_z(p)}}{k!} \right) \right] \]
\[ = \max_{p \in [p^{min}, p^{max}]} 0.92 p \left[ c + \sum_{k=0}^{c} (k - c) \left( \frac{(50 - 2p)^k e^{-50+2p}}{k!} \right) \right], \]

\[ V_1(c) = \max_{p \in [p^{min}, p^{max}]} E[v(c, p) + V_2(c - (1 - q_i) \min\{d(p), c\})] \]
\[ = \max_{p \in [p^{min}, p^{max}]} (1 - \alpha q_i) E[p \min\{d(p), c\}] + E[V_2(c - (1 - q_i) \min\{d(p), c\})] \]
\[ = \max_{p \in [p^{min}, p^{max}]} (1 - \alpha q_i) p \left[ c + \sum_{k=0}^{c} (k - c) \left( \frac{(\mu_i(p))^k e^{-\mu_i(p)}}{k!} \right) \right] \]
\[ + \left[ V_2(q_i c) + \sum_{k=0}^{c} (V_2(c - (1 - q_i) k) - V_2(q_i c)) \left( \frac{(\mu_i(p))^k e^{-\mu_i(p)}}{k!} \right) \right] \]
\[ = \max_{p \in [p^{min}, p^{max}]} (1 - \alpha q_i) c + V_2(q_i c) \]
\[ + \sum_{k=0}^{c} \left[ (1 - \alpha q_i)(k - c) p + V_2(c - (1 - q_i) k) - V_2(q_i c) \right] \left( \frac{(\mu_i(p))^k e^{-\mu_i(p)}}{k!} \right) \]
\[ = \max_{p \in [p^{min}, p^{max}]} 0.92 c p + V_2(0.1 c) \]
\[ + \sum_{k=0}^{c} \left[ 0.92(k - c) p + V_2(c - 0.9 k) - V_2(0.1 c) \right] \left( \frac{(30 - p)^k e^{-30+p}}{k!} \right). \]

The corresponding total optimal expected revenue and products’ price of stage 1 are calculated assuming that the numbers of the perishable products are 10, 15, 20, 30 and 40 respective.

As can be seen from Fig 1, the optimal revenue is increased by the products capacity increasing, \( V_1(c) \) is concave in \( c \), and from Fig 2, the optimal price is decreased by the products capacity increasing.
4. Conclusions

This research presents a stochastic dynamic pricing and allocation model for perishable products reservations on a future given day considering no-shows and cancelations under an uncertain environment. Customers are assumed to be price sensitive, the price is changed from period to period in reservation horizon, and the expected demand in each period is different. We consider perishable products to have only one resource, and every reservation period may cover several days.

Some respects can be research further, however. Such as, in this research, we give the optimal pricing policy only considering the expected revenue based one product, but customers often order more than one product. And that the no-shows and cancelations of previous period are considered in its following period should be research further.
5. References