Performance Analysis of a QR Decomposition Algorithm Employing Rearrangement of Antenna Layers on V-BLAST Systems

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Abstract

In a Rayleigh fading environment, the paper firstly introduces system model and the vertical Bell laboratories layered space-time (V-BLAST) systems structure. Several popular decoding algorithms for V-BLAST are carried on the analysis comparison, such as zero-forcing (ZF), minimum mean squared error (MMSE), QR decomposition, and maximum likelihood (ML) etc. On this basis, the paper focuses on QR decomposition research, analyzes and summarizes the QR decomposition algorithm and iterative algorithm. We draw a conclusion that the key of improving QR scheme performance is to choose the upper triangular matrix $R$. From intuitionistic point of view, three selection criterions are given, such as rearranging matrix $H$ according to row vectors, the maximum element $r(N_l, N_t)$ of the matrix $R$, and the maximum diagonal squared Frobenius norms (SFN) in $R$. The simulation results indicate that the selection rule of the maximum $r(N_l, N_t)$ can obviously improve the performance of the first decoded layer. For a $4Tx\times4Rx$ MIMO V-BLAST system, the simulation results show that the V-BLAST system performance is to be improved further when we combined the maximum $r(4, 4)$ criterion with the iterative algorithm.

Keywords: V-BLAST, QR Decomposition, Iterative Algorithm, Bit Error Rate

1. Introduction

In a flat Rayleigh fading environment, multiple antenna systems provide an enormous increase in capacity compared to single antenna systems [1–3]. To take advantage of multiple antennas, space-time codes (STC) have been introduced to use space as the second dimension of coding. In multiple input multiple output (MIMO) systems with $N_t$ transmit and $N_r$ receive antennas, although space-time trellis codes (STTCs) [4] and space-time block codes (STBCs) [5] can achieve a diversity order of $N_t \times N_r$ over quasi-static fading channels, rendering such codes very effective a spatial rate of unity or less (in terms of the number of symbols per channel use) without offering flexibility in trading diversity for rate. Providing such a trade-off is essential to accommodate a wide range of wireless applications, especially those applications that have high data rate requirements.

The notion of layered space-time (LST) coding, first introduced by Foschini [6], has emerged since then as a powerful architecture suitable for applications with high data rates. Several LST coding architecture exist, including horizontal Bell laboratories layered space-time (H-BLAST), vertical BLAST (V-BLAST), and diagonal BLAST (D-BLAST) architectures[7]-[13]. Other LST schemes also exist, including the multilayered space-time coding scheme proposed by reference [7]-[8]. These schemes combine channel coding and LST coding, resulting in a trade-off between diversity and rate. In this paper, we detailed study the V-BLAST because of its importance and easy implementation, we describe its encoding mechanism, and highlight its merits relative to the other architectures. We also compare in brief bit error rate (BER) performance of the detection algorithms used for V-BLAST scheme, e.g., zero-forcing (ZF), minimum mean squared error (MMSE), QR decomposition, and maximum likelihood (ML) detection criteria [8]-[9]. Although both ZF and MMSE detection algorithms can suppress completely the interferences from all other launched antennas, and simplify decoding for each layer separated symbols, but they bring a similar problem that the decoder at the receiver must perform to operate complex channel matrix inverse. While the interferences are suppressed fully and the signals at every layer are separated, the noises at the receiver are also magnified, especially for the ZF method. In addition, using this suppression technique, we unable to make full use of the antenna spatial diversity characteristics at every decoded layer, they only achieve $N_r-N_t+1$ diversity order. And the QR scheme is quite the opposite, it employs matrix decomposition method with lower computing complexity, in addition to the first detected layer, interference
cancellation and suppression is used at other layers, it makes full use of the spatial diversity ability, its performance is slightly better than the MMSE method.

QR decomposition is a practical and promising technology, this paper will study mainly QR algorithm. At present, the QR detection schemes are widely used in V-BLAST system research [14]–[16]. Ref. [14] for the first time puts forward QR decomposition to detect symbols, which requires only a fraction of computational effort compared to the standard decoding algorithm requiring the multiple calculation of the pseudo inverse of the channel matrix. Ref.[15] introduced iterative QR detection for BLAST. Ref.[16] presents an in-depth analysis of the zero forcing (ZF) and minimum mean squared error (MMSE) equalizers applied to wireless multiinput multioutput (MIMO) systems with no fewer receive than transmit antennas. Other relevant documents such as [17]–[22].

The organization of the paper is as follows. Section 2 a MIMO channel model is presented, and common decoding algorithms are compared. Section 3 describes QR decomposition and its iterative algorithm. Improved algorithms, simulation results, BER performance and analysis are demonstrated in Section 4. Finally, we have drawn conclusions in Section 5.

2. Common algorithm comparison

2.1. System and channel model

![MIMO system diagram](image)

Figure 1. The multiple input multiple output (MIMO) system diagram

We assume that there are \( N_t \) transmit and \( N_r \) receive antennas as illustrated in Figure 1, and that there is no intersymbol interference (i.e., the sub-channels are flat fading). The input-output relationship of the MIMO channel is given by

\[
\begin{bmatrix}
  y_1 \\
  y_2 \\
  \vdots \\
  y_{N_r}
\end{bmatrix} =
\begin{bmatrix}
  h_{11} & h_{12} & \cdots & h_{1N_r} \\
  h_{21} & h_{22} & \cdots & h_{2N_r} \\
  \vdots & \vdots & \ddots & \vdots \\
  h_{N_t,1} & h_{N_t,2} & \cdots & h_{N_t,N_r}
\end{bmatrix}
\begin{bmatrix}
  x_1 \\
  x_2 \\
  \vdots \\
  x_{N_t}
\end{bmatrix} +
\begin{bmatrix}
  n_1 \\
  n_2 \\
  \vdots \\
  n_{N_r}
\end{bmatrix},
\] (1)

or matrix form as

\[
y = \mathbf{H} \mathbf{x} + \mathbf{n},
\] (2)

where \( y=[y_1(k), y_2(k), \cdots, y_{N_r}(k)]^T \) is the \( N_r \times 1 \) vector of received signals at \( k \) time. \( \mathbf{x}=[x_1(k), x_2(k), \cdots, x_{N_t}(k)]^T \) is the \( N_t \times 1 \) vector of transmitted signals (whose components are complex numbers), the signal vector satisfies the power constraint \( E[\mathbf{x} \mathbf{x}^H] \leq 1 \). \( \mathbf{H} \) is the \( N_r \times N_t \) matrix, its element \( [h_{ij}] \) denotes the channel gains for the \( j \)th transmit and the \( i \)th receive antenna pair, it is a complex Gaussian random variable with zero mean and variance 0.5 per dimension and for different uses of the channel are independent. \( \mathbf{n}=[n_1(k), n_2(k), \cdots, n_{N_r}(k)]^T \) is the \( N_r \times 1 \) vector of complex Gaussian noise terms, the noise terms are independent samples of circularly symmetric zero-mean complex Gaussian random variables with variance \( N_0/2 \) per dimension, \( N_0 \) is single-sided noise power spectral density.
2.2. V-BLAST architecture

The V-BLAST encoder is depicted in Figure 2. As shown in the figure, the message bit stream is demultiplexed into $N_t$ parallel substreams. Each substream is modulated using a $M$-ary complex constellation, $M=2^b$, interleaved and then assigned to transmit antennas. As such, the number of layers is $N_t$ and the spatial rate is $bN_t$. Since each layer is associated with a fixed transmit antenna, this architecture can accommodate applications with possibly different data rates and/or different users. The spatial diversity achieved by this scheme varies between one and $N_r$, depending on the detection scheme employed at the receiver. For instance, when interference cancellation and suppression is used, the first layer detected will have a spatial diversity of $N_r-N_t+1$ because the other layers are suppressed where they treated as interference [13]. The last layer detected, on the other hand, will have a spatial diversity of $N_r$ since the $N_t-1$ previously detected layers are subtracted from the last layer, i.e., there is no suppression but rather cancellation.

![Figure 2. N_t=4 transmit antennas, the V-BLAST transmission data model](image)

2.3. Analysis comparison of decoding algorithms

The bit error rate performance results for the V-BLAST scheme are shown in Figure 3 for the following detection criteria: zero forcing (ZF), minimum mean squared error (MMSE), QR decomposition (QR), iterative QR and maximum likelihood (ML). From the figure, we can see that the optimal detector for the V-BLAST signals is the ML detector, however, the complexity of the detector grows exponentially in the number of transmit antennas and the number of states of the digital modulation constellation. When interleaving is employed, the complexity grows even further. Even for a small number of transmit antennas, the associated complexity is still too high, rendering this
detection approach not an option. Other less complex but suboptimal approaches are commonly used for detecting V-BLAST signals, including the detection algorithms based on the ZF, the MMSE, and the QR decomposition criteria. In the ZF criterion, when a layer is detected, the interference coming from undetected layers is suppressed, whereas in the MMSE criterion, a compromise between interference suppression and noise reduction is achieved. Therefore, the performance of the MMSE detection criterion is better than that of the ZF detection criterion. While both detection approaches are asymptotically equivalent, the ZF approach is less practical than the MMSE approach because the complete interference suppression achieved by ZF comes at the expense of enhancing the noise power, which leads to performance degradation. Another difference between the two schemes is that the constraint $N_r \geq N_t$ that is required for the ZF detector can be relaxed for the MMSE detector. For both detection schemes, interference suppression can be combined with interference cancellation to achieve further improvement in performance. In addition, we can also see that additional performance improvements can be obtained when QR decomposition algorithm with interference cancellation is used. However, the three kinds of results are still far away from the ML detection. But if one introduces the iterative loop algorithm in the QR scheme, the V-BLAST system performance will be greatly improved[15].

3. QR Decomposition algorithm

3.1. QR algorithm

Assuming quasi-static fading and that $N_r \geq N_t$, $H$ can be expressed using the QR factorization principle as $H=QR$, where $R$ is an $N_t \times N_t$ upper triangular matrix and $Q$ is an $N_r \times N_t$ unitary matrix. The rows of $Q$ are mutually orthogonal. Consequently, $Q$ has the property that $QQ^H=I_{N_t}$, where $I_{N_t}$ is the $N_t \times N_t$ identity matrix. There are several fast algorithms available in the literature [23] that can be used to compute these vectors. If $H$ is nonsingular, which is the case for Rayleigh fading channels (with probability 1), the diagonal elements of $R$ are all positive. Representing $H$ as such is essential in suppressing inter-layer interference. To elaborate, let us left-multiply $y$ in (2) by $Q^H$. Let $Y$ denote the matrix resulting from this multiplication, that is,

$$Y=Q^Hy=Q^Hhx+Q^Hn$$  \hspace{1cm} (3)

$$=Rx+N,$$  \hspace{1cm} (4)

where $N=Q^Hn$. Equation (4) is obtained by plugging the expression $H=QR$ into (3) and knowing that $QQ^H=I_{N_t}$.

The $i$th element of $Y$, denoted by $Y_i(k)$, represents the received signal from the $i$th receive antenna at time $k$, which can be expressed as

$$Y_i(k)=\sum_{j=1}^{N_r} r_{j,i} x_j(k) + N_i(k)$$  \hspace{1cm} (5)

$$=r_{i,i} x_i(k) + \sum_{j=i+1}^{N_r} r_{j,i} x_j(k) + N_i(k)$$  \hspace{1cm} (6)

where $r_{j,i}$ is the $(j,i)$th element of $R$, at $k$ time, $x_i(k)$ is the $j$th element of $x$, and $N_i(k)$ is the $i$th element of $N$. The first term on the right side in (6) is the desired symbol, and the second term is interference. The lower limit on $j$ in (5) is $i$ because $R$ is an upper triangular, i.e., $r_{j,i}=0$ for $j=1,2,\ldots,i-1$. It is clear from (6) that the interference coming from layers 1, $\ldots$, $i-1$ has been suppressed. As for the interference coming from the remaining layers, since these layers have already been detected, it can be cancelled easily.

From the above discussion, it is clear that the detection should start with layer $i=N_t$. In this case, $Y_{N_t}(k)$ is expressed as
\[ Y_{N_t}(k) = r_{N_t,N_t} x_{N_t}(k) + N_{N_t}(k) \]  

(7)

It is clear from (7) that there is no interference from other layers, and hence an estimate of the signal corresponding to this layer, denoted as \( \hat{x}_{N_t}(k) \), can be obtained. Next, layer \( i=N_t-1 \) is detected and this involves suppressing the interference coming from layers \( j=1,2,\ldots, N_t-2 \) and canceling the interference coming from layer \( N_t \). This process continues until the last layer is detected. In general, when layer \( i \) is to be detected, the interference coming from layers \( j=1,2,\ldots, N_t-1 \) is suppressed and the interference coming from layer \( N_t \) is cancelled. The interference to be cancelled from layer \( i \) can be expressed as

\[ N_j(k) = \sum_{j=i+1}^{N_t} r_j \hat{x}_j(k). \]

As such, the decision statistics for the \( i \)th layer, namely \( \hat{Y}_i(k) \), can be written as

\[ \hat{Y}_i(k) = r_i x_i(k) + \sum_{j=i+1}^{N_t} r_j [x_j(k) - \hat{x}_j(k)] + N_i(k). \]  

(8)

The expression in (8) suggests that, when the decisions for all previously detected layers are all correct, the next layer to be detected is interference-free, although in practice this assumption is somewhat optimistic. A summary of QR with interference cancellation algorithm for V-BLAST systems is given in Table 1.

**Table 1.** The QR algorithm for the V-BLAST scheme

| Decompose $H$ according to (3) |
| Compute $Y$ according to (4) |
| Detect layer $N_t$ according to (7) |
| Do $\hat{x}_{N_t} = \text{ML}(Y_{N_t}/x_{N_t})$ decision |
| Set $i = (N_t-1)$ |
| While $i \geq 1$ |
| Subtract the interference from layer $i$ according to (8) |
| Detect layer $i$ and obtain decisions using defined by (8) |
| $i = i-1$ |
| End of the while loop |

### 3.2. QR iterative algorithm

The above know, on the QR detecting algorithm, the first detected symbol is the $N_t$th layer, the detected symbol \( \hat{x}_{N_t}(k) \) is subtracted from all other ($N_t-1$) layers. These operations are repeated for the lower layers, finishing with layer 1, which, assuming that all symbols at previous layers have been detected correctly, will be free from interference. The decision statistics from the detector at each layer is passed to a decision making device in a V-BLAST systems. The drawback of the first layer detected symbol is that achievable diversity order is $(N_r-N_t+1)$. If the interference suppression starts at layer $N_t$, then at this layer $(N_t-1)$ interferers need to be suppressed. Assuming that $N_t \geq N_r$, the diversity order in this layer is $(N_r-N_t+1)$. In the first layer, there are no interferers to be suppressed, so the diversity order is $N_r$.

After the QR decomposition, the decision symbols of the final detected layer (equivalent to the first layer) can achieve diversity order $N_r$, and they have higher accuracy, the bit error rate is lower. The basic idea of the iterative algorithm is that the final detected layer symbols are moved down to the first detected layer (the $N_t$th layer), other layers also in turn are moved down to next one floor, namely, $L_1 \rightarrow L_{N_t}, L_{N_t} \rightarrow L_{N_t-1}, \ldots, L_2 \rightarrow L_1$. Note that channel matrix $H$ remains constant during a whole iterative cycle. The QR iterative algorithm is summarized in Table 2.
4. Improved QR scheme and results analysis

By examining (8) more closely, we can see the level of data reliability differs from one layer to the other. Specifically, the first detected layer has the lowest reliability, whereas the last detected layer has the best reliability. As mentioned before, in the former case, the diversity order achieved is \(N_r - N_t + 1\) because the contribution from all other layers is suppressed at the first detected layer. On the other hand, the diversity order of the last detected layer is \(N_r\) since the interference coming from all previously detected layers is cancelled and not suppressed. In general, the diversity order of the \(i\)th layer is \((N_r - N_t + i)\) [13]. In the above analysis, we assume that all symbols at previous layers have been detected correctly, therefore, the QR scheme usually exists error propagation. It is thus clear that the first detected layer of the accuracy will determine the system performance. From the intuitive point of view three methods are put forward, they are the sorting row vector of the matrix \(H\), the maximum diagonal squared Frobenius norms (SFN) of the matrix \(R\) and the maximum \(r(N_r, N_t)\) of the matrix \(R\) three kinds of cases, respectively.

4.1. Three QR detection schemes

4.1.1. Rearranging matrix \(H\) according to row vectors

This approach is to sort \(H\) according to each row SFN, the row with the highest SFN becomes the \(N_t\)th row of \(H\) and the row with the lowest SFN becomes the first row of \(H\), the rest of the QR interference cancellation proceeds as outlined in the previous section. It is clear that different order for channel matrix \(H\) will correspond to different detected layer, for example, without sorting, the first detected layer corresponds to the signal transmitted from the \(N_t\)th transmit antenna and the last detected layer corresponds to the signal transmitted from the first transmit antenna, whereas, with sorting, the first detected layer could correspond to any of the transmitted signal antennas. In fact, the order of the detected layers follows the order of the rows of the sorted \(H\).

**Table 3. the QR algorithm by sorting row vectors for \(H\)**

| for \(k = N_r - 1:2\) |
| for \(l = k-1: -1:1\) |
| if \(\text{norm}(H(k,:)) > \text{norm}(H(l,:))\) |
| \(H(k,:) \leftrightarrow H(l,:); \ H(:,k) \leftrightarrow \ H(:,l);\) |
| end the if loop; |
| end the for \(l\) loop; |
| end the for \(k\) loop; |
| do the QR algorithm such as Table 1 |

4.1.2. Selection criteria based on the maximum element \(r(N_r, N_t)\)

Transform the channel matrix \(H\) through the change of different permutation between different layers, these channel matrixes are resolved by QR decomposition, the resulting matrix with maximum element \(r(N_r, N_t)\) of the upper triangular matrix \(R\) is selected. A summary of the selection algorithm for the maximum \(r(N_r, N_t)\) is given in Table 4.
4.1.3. Selection criteria based on maximum diagonal SFN by matrix $R$

The same as the subsection 4.1.3, transform the channel matrix $H$ by the different permutations between the layers, these channel matrixes are resolved by QR decomposition, we select the resulting matrix $H$ with maximum diagonal SFN $\sum N_i r_{i,i}$ for $R$. A summary of the selection algorithm is given in Table 5.

4.2. Performance results

In the frequency nonselective flat Rayleigh fading channel, for a 4Tx×4Rx MIMO system employing V-BLAST encoding scheme and binary phase shift-keying (BPSK) digital modulation. At the same time, to facilitate to compare with the single antenna systems, we assume that all transmit antennas are equal power launch, and total emission power has been normalized. Further assumption that during the whole layer space-time encoding and decoding the complex channel gains between transmit and receive antennas are constant. As above analysis, the drawback for QR decomposition is the existing of error propagation. Therefore, the decoding accuracy of the first layer with minimum diversity order (here diversity order is 1) will determine system performance. Aimed at upper triangular matrix $R$, in order to compare performance of three selection criterions described in the subsection 4.1, the Figure 4 depicts three bit error rate (BER) performance curves of the initial layer decoded symbols employing

**Table 4. the QR algorithm by Maximum $r (N_t, N_t)$**

<table>
<thead>
<tr>
<th>set all the permutations of matrix $H$</th>
</tr>
</thead>
<tbody>
<tr>
<td>for $i=1$ to number of permutation</td>
</tr>
<tr>
<td>do QR decomposition for $H$</td>
</tr>
<tr>
<td>select maximum $r (N_t, N_t)$</td>
</tr>
<tr>
<td>end of the for loop</td>
</tr>
<tr>
<td>do the QR algorithm such as Table 1</td>
</tr>
</tbody>
</table>

**Table 5. the QR algorithm by Maximum Diagonal SFN**

<table>
<thead>
<tr>
<th>set all the permutations of matrix $H$</th>
</tr>
</thead>
<tbody>
<tr>
<td>for $i=1$ to number of permutation</td>
</tr>
<tr>
<td>do QR decomposition for $H$</td>
</tr>
<tr>
<td>select maximum diagonal SFN for matrix $R$</td>
</tr>
<tr>
<td>end of the for loop</td>
</tr>
<tr>
<td>do the QR algorithm such as Table 1</td>
</tr>
</tbody>
</table>

**Figure 4. Three kinds of different upper triangle matrixes $R$, the BER performance comparison curves of decoding symbols of the first layer.**
the common QR algorithm, sorted row vectors for the matrix $H$, and the maximum $r(4,4)$ (denoted by max R44 in the figure 4, hereinafter the same), respectively. Simulation results show that the performance of both the sort QR decomposition and ordinary QR is almost no difference. It also shows that from statistical perspective the sorting channel matrix $H$ has no direct relation with the maximum $r(4,4)$. And the decision performance using the decode criterion of the maximum $r(4,4)$ is far better than ordinary QR decomposition. When error probability $P_e=2\times10^{-2}$, about 4 dB BER performance is improved.

![Figure 5. Performance comparison of various kinds QR algorithm.](image)

![Figure 6. QR algorithm performance curve charts for loop iterative + the maximum R(4,4).](image)

Based on the selection criterion of maximum diagonal SFN and the maximum $r(4,4)$ for the upper triangular Matrix $R$ using QR decomposition, Figure 5 gives their BER performance comparison. The simulation results show that the maximum $r(4,4)$ performs better than the ordinary QR decomposition, this further indicates that the error propagation will have a great influence on performance of QR scheme, therefore, ensure the decoding accuracy of the first floor is critical. Figure 5 shows that the performance of maximum diagonal SFN didn't expect good, instead more bad, because the selection rule can ensure that the diagonal SFN is always the greatest, but it does not guarantee that both $r(4,4)$ and $r(3,3)$ are enough big, thus the whole system performance may become worse. The graph also shows that the BER performance of once iterative QR algorithm is far better than the usual QR decomposition and the maximum $r(4,4)$, this is mainly due to the first detected floor of iterative algorithm use directly the decision symbols of the final detected layer for the QR scheme, which the
diversity order obtained is 4. Therefore, the iterative algorithm is a very promising V-BLAST decoding algorithm. The paper have combined the maximum $r(4,4)$ criterion and the iterative algorithm, the Figure 6 gives their BER performance curves versus signal-to-noise ratio ($E_b/N_0$). The simulation results show that the combination (once loop iteration and the maximum $r(4,4)$) performs better than the iterative algorithm, when error probability $P_e=2\times10^{-2}$, compared to common QR scheme, about 7 dB system performance is improved. When adding iterative times, one can foresee, their performance will be further improved.

5. Conclusions

This paper studies mainly a 4 transmit and 4 receive antennas (4Tx×4Rx) V-BLAST communication system, the comprehensive analysis and comparison of several decoding algorithms of the layered space-time structure has been discussed. We consider synthetically system error performance and decoding of computational complexity, We think the QR decomposition algorithm has its advantages. To further improve the performance of QR scheme of the V-BLAST systems, deal with the key to the problem is how to increase the equivalent channel gains of the first detected layer. By choosing a new permutation of channel matrix $H$, we can achieve the largest $r(4,4)$ in the matrix $R$ after QR decomposition. From intuitionistic point of view, this paper gives three selection criterions, such as rearranging matrix $H$ according to row vectors, the maximum element $r(N_t, N_t)$ of the matrix $R$, and the maximum diagonal squared Frobenius norms (SFN) in $R$. The simulation results show that the QR scheme using the maximum element $r(N_t, N_t)$ can obviously improve the corresponding system performance, combination the iterative QR algorithm with the maximum $r(N_t, N_t)$ criterion can also get to further improve the performance. the shortcomings of the maximum $r(N_t, N_t)$ criterion is how fast to get the corresponding channel matrix $H$, this is a future challenges.

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