Instantaneous Frequency Estimation Algorithm of Non-linear Frequency Modulation Signal in Low SNR Based on Generalized S-transform

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Abstract

The Nonlinear Frequency Modulation (NLFM) signals have been gradually used in new radar systems, however its instantaneous frequency (IF) estimation is still a difficult problem. This paper introduces the generalized S-transform to NLFM signals processing fields and proposes a new IF estimation algorithm of NLFM signals in low SNR based on generalized S-transform. Firstly, the time-frequency distribution (TFD) of NLFM signals in low SNR is got by generalized S-transform. Secondly, a time-frequency (TF) filtering processing is executed to the TFD of NLFM signals and the noise is restrained. Finally the TFD spectrum ridges are extracted as IF curves estimation from the filtered TFD of NLFM signals. Simulation results demonstrate that the IF estimation algorithm proposed in this paper has better performances in IF estimating of NLFM signal in low SNR than other existing algorithms.

Keywords: Signal Processing, Nonlinear Frequency Modulation Signal, Instantaneous Frequency Estimation, Generalized S-Transform, Low SNR.

1. Introduction

The Nonlinear Frequency Modulation (NLFM) signals have superiorities of small mismatch losses and low probability intercept etc., which making it being gradually used in new radar systems. The instantaneous frequency (IF) is the most basic and important feature in representation of NLFM signals, therefore IF estimation is a very important work and it has become a research hotspot in NLFM signal processing. The existing IF estimation methods of NLFM signals focus on the complex argument distributions algorithm [1], the fractional Fourier transform (FRFT) algorithm [2], the phase difference algorithm [3], the quadratic time-frequency distribution (QTFD) algorithm[4], the spline based on Chirp atoms method [5] and the pseudo Wigner-Ville distribution (PWVD) algorithms [6]. However all these algorithms have no satisfactory performances to multicomponent NLFM signal in low SNR case. The S-transform proposed by Stockwell, et al.[7] is a linear time-frequency (TF) representation which has very good performance of TF concentration and has no cross-terms interference; therefore it can be applied to nonstationary signal processing. Gao. et al.[8] added two adjustment parameters to S-transform and proposed the generalized S-transform which improves the TF concentration on the basis of remaining the features of linear characteristic, etc. Wang et al. [9] combined the reassigned generalized S-transform and Hough transform and proposed a multicomponent LFM signal detection and parameter estimation algorithm. This paper introduce the generalized S-transform to NLFM signals analysis field and proposed an IF estimation algorithm of NLFM signals in low SNR, which solved the IF estimation problem of multicomponent NLFM signals in low SNR. Simulation results demonstrate that the IF estimation algorithm proposed in this paper has better performances in IF parameter estimating of low SNR NLFM signals than other existing algorithms, which provide a novel way to parameters estimation of multicomponent NLFM signals in low SNR case.
2. Generalized S-Transform and TF filtering

2.1. Generalized S-Transform of multicomponent NLFM signals

By combining the STFT and wavelet, the S-transform and its invert transform of a time series derived by Stockwell, et al. in [7] are defined as:

\[
S(\tau, f) = \int_{-\infty}^{\infty} u(t) \left| \frac{1}{\sqrt{2\pi}} e^{-\frac{(t-\tau)^2}{2}} \right| e^{-j2\pi ft} \, dt
\]

(1)

\[
u(t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S(\tau, f) d\tau \, e^{j2\pi ft} \, df
\]

(2)

where \(S(\tau, f)\) is S-transform of \(u(t)\), \(f\) is frequency, \(\tau\) is the center of time window. To improve the TF concentration of S-transform, Pinnegar. [10], Chen et al. [11] introduced adjust factors and proposed the generalized S-transform defined as:

\[
S(\tau, f) = \int_{-\infty}^{\infty} u(t) \left| \frac{1}{\sqrt{2\pi}} \left| f \right|^{\frac{p}{2}} \right| e^{-\frac{1}{2} \left( \frac{t-\tau}{\lambda_{GS}} \right)^2} e^{-j2\pi ft} \, dt, \quad p \in [1/2, 3/2]
\]

(3)

where \(\lambda_{GS}\) and \(p\) are adjustment parameters and their values are depended on specific application experientially. From [10] and [11] we can know that generally the value of \(p\) is from 0.5 to 1.5. If the value of \(p\) is defined, the window width of the generalized S-transform is broadening with the value of \(\lambda_{GS}\) is increasing; when \(\lambda_{GS}\) is decreasing (tend to 0), the width of window is narrowing also. In order to get better time resolution, smaller \(\lambda_{GS}\) should be chosen however the frequency resolution will be worse. If \(\lambda_{GS} = p = 1\) it converts to basic S-transform. Therefore we can get better time-frequency resolution through choosing proper values of \(p\) and \(\lambda_{GS}\), which enhance the adaptivity of generalized S-transform in nonstationary signal processing. B. Biswal [12] proposed a TF analysis and FPGA implementation method of modified S-transform algorithm for denoising, which promote the application of S-transform to signal processing.

For a multicomponent NLFM signal \(u(t)\) which contains multiple components of \(u_1(t), u_2(t), ..., u_n(t)\) and its express is

\[
u(t) = \sum_{i=1}^{n} u_i(t) = \sum_{i=1}^{n} a_i \exp[j\phi_i(t)]
\]

(4)

where \(a_i\) and \(\phi_i(t)\) are amplitude and instantaneous phase (IP) of \(u_i(t)\). According to the definition and linear characteristic of generalized S-transform we can get the generalized S-transform TFD of \(u(t)\) as:
where $GS_n(\tau, f)$ is the generalized S-transform of $u(t)$, $w(\tau - t)$ is window function of generalized S-transform.

Figure 1 shows the time waveform and TFD of Wigner-Ville distribution (WVD), PWVD and generalized S-transform of a two-component NLFM signal respectively.
theory of TF concentration we can know that the TFD regions $GS_n(n,k)$ occupied by effective signal can be extracted from $GS_n(n,k)$ by setting a suitable threshold $\alpha$:

$$GS_n(n,k) = GS_n(n,k) \text{ind}_n(n,k).$$  \hspace{1cm} (6)

$$\text{ind}_n(n,k) = \begin{cases} 1, & GS_n(n,k) \geq \alpha \\ 0, & GS_n(n,k) < \alpha \end{cases} \quad (n,k) \in R.$$  \hspace{1cm} (7)

where $\text{ind}_n(n,k)$ is the TF pass region of the $i$th component of effective signal, the setting method of threshold $\alpha$ is given in [13]. After getting all the $GS_n(n,k)$ we can remove the TFD of noise outside of $GS_n(n,k)$. Based on the TFD otherness of effective signal and noise, we exploit an adaptive TF filtering preprocessing by the TF spectrum of $u(t)$ to restrain noise in the regions of $GS_n(n,k)$ and the major steps as

$$H_n(n,k) = \frac{|GS_n(n,k)|}{\alpha + |GS_n(n,k)|}.$$  \hspace{1cm} (8)

$$GS_n(n,k) = GS_n(n,k)H_n(n,k).$$  \hspace{1cm} (9)

where $H_n(n,k)$ is the TF filtering factor constructed by the TFD features of $u(t)$, $GS_n(n,k)$ is the discrete TFD of effective signal obtained by two steps TF filtering processing, $\alpha$ is the threshold and the value range of TF filtering factor result is from 0 to 1.

3. IF Estimation of Multicomponent NLFM Signal

3.1. IF determination methods to multicomponent NLFM signals

The ridges of TFD have characteristics similar to the local modulus maxima, which not only contains characteristics of signal but also depicts energy concentration regions of signal. If the regions and the ridges can be described efficiently then its IF features can be extracted effectively. Supposing a generalized S-transform TFD of $GS_n(n,k)$ and its expression as

$$GS_n(\tau, f) = \int_{-\infty}^{\infty} u_n(t)w(\tau - t, f)e^{j2\pi f \tau}dt = \int_{-\infty}^{\infty} e^{j\varphi(t)}w(\tau - t, f)e^{j2\pi f \tau}dt$$  \hspace{1cm} (10)

To any component of $u_n(t)$, its IF is defined as the IP derivative on the time, that is, $IF(t) = \frac{d\varphi(t)}{dt}$. According to the definition of IF, peaks of $GS_n(n,k)$ should be appeared at the positions of IF, thus the ridges of TFD contain parameters of signals: the fluctuations are corresponding to the variation of spectrum amplitude and the positions of ridges corresponding to the variation of IF. If the ridges of each region occupied by effective signal component on TFD plane can be extracted (that is, estimate the IF on the maximization rule of TFD), they are just the IF of the component:

$$IF(n) = \arg \max_{p \in Q} GS_n(n,k).$$  \hspace{1cm} (11)
where \( Q = \{ \text{IF} \mid 0 \leq \text{IF} \leq f_s / 2 \} \), \( f_s \) is sampling frequency equation 11 indicates that the amplitudes of TF representing for the IF corresponding to the local maxima points on the TF plane, the IF of each component can be estimated by extracting the ridges constructed by these local maxima points.

### 3.2. Ridges extracting method of TFD

In this paper we use the ridges exacting method presented in [14-15]. Suppose a filtered TFD \( G_S(n,k) \) is a matrix of \( K \times L \), to any position of \((n,k)\), its amplitude is \( M(n,k) \), \( n \in [1,K] \), \( k \in [1,L] \), the major steps of ridges extracting as follows: firstly, traverse all the regions of effective signal and exact the approximate ridges by climbing algorithm, the extracted ridges points are saved in a measure matrix \( D \); secondly, to each element of \( D \), searching its optimal neighboring points and connecting them to a line link, then repeating the operation above until all the elements of \( D \) are all in the link; thirdly, computing the energy of each ridge line then getting ride of the ridges does not occupy superiorly energy based on threshold rule, the final ridges will be obtained and these ridge lines is just the IF curve.

### 3.3. Algorithm flow of IF estimation

The main steps of IF estimation algorithm of multicomponent low SNR NFM signals based on generalized S-transform proposed in this paper as:

**Step 1.** Getting the discrete TFD \( G_S(n,k) \) of low SNR multicomponent NFM signal \( u(t) \) by generalized S-transform.

**Step 2.** Extracting the TF regions occupied by effective signal and removing all the other TFD beside these regions according to the different characteristics of effective signal and noise.

**Step 3.** Constructing TF filtering factor to filter the regions of effective signals to restrain the TFD of noise in these regions.

**Step 4.** Obtaining the ridge lines of each regions of effective signal by ridge extraction algorithm and the ridge lines are just the IF curve estimated by the extracted ridge lines of TF regions.

### 4. Simulation results and performance analysis

#### 4.1. Simulation results

To evaluate the performance of the proposed algorithm, two multicomponent NLFM signals are used in this paper. Firstly we use a NLFM signal contains two sinusoidal frequency modulation components which is generated by the fmsin function of Matlab software with its main parameters as follows: the lengths \( N_1 = N_2 = 512 \), the amplitudes \( A_1 = A_2 = 1 \), the minimum normalized frequencies \( f_{\text{NORMIN}} = 0.1 \), \( f_{\text{NORMIN2}} = 0.22 \); the maximum normalized frequencies \( f_{\text{NORMAX}} = 0.25 \), \( f_{\text{NORMAX2}} = 0.4 \); the cycles \( T_1 = T_2 = 500 \). After adding zero mean white Gaussian noise, the SNR is \(-5\) and the generalized S-transform TFD is shown as figure 2.
Figure 2. The TFD of the two-component NLFM signal with noise. (a) Planar image of generalized S-transform TFD; (b) Three-dimensional graphic of generalized S-transform TFD.

Figure 2 shows that the TFD of NLFM signal is interfered seriously in low SNR, if we estimate the IF by extracting spectrum ridges from this TFD directly; the estimated error will be great. Furthermore, a mass of false peaks emerge and the parameters estimation performance decrease seriously. To improve the IF estimation performance of NLFM signal in low SNR, this paper execute a TF filtering processing to the noisy signal to remove the noise components outside the TF regions of effective signal at first, and then construct a TF filtering factor to remain the noise components inside the TF regions of effective signal. The generalized S-transform TFD of filtered signal is shown as figure 3, the SNR of filtered signal is enhanced to 7.29 and the elevation amount is reach to 12.29.

Figure 3. The generalized S-transform TFD of filtered NLFM signal.

To the filtered TFD of NLFM signal, we extract the ridges of TF regions occupied by effective signal components as the estimated IF waveform (denote: due to the frequency adopted in sinusoidal
frequency modulation signal is normalized frequency, here the extracted frequency is normalized also) is shown as figure 4.

![Figure 4](image)

**Figure 4.** The ideal and estimated IF waveforms of NLFM signal.

From figure 4 we can know that even in the low SNR case, the IF waveform estimated by the algorithm proposed in this paper fits the true IF waveform very well, which indicates that the IF estimation algorithm still has good IF estimation performance to multicomponent NLFM signal even in low SNR.

Secondly we use a NLFM signal combined with two components of a LFM signal and a parabolic FM signal, and its TFD is shown as figure 5. Then we use the proposed IF estimation algorithm and the estimation result is shown as figure 6.

![Figure 5](image)

**Figure 5.** The generalized S-transform TFD of NLFM signal.
Figure 6. The ideal and estimated IF waveforms of NLFM signal

From figure 6 we can see that the IF waveform estimated by the proposed algorithm fits the true IF waveform very well also, which indicates that the proposed IF estimation algorithm still has well IF estimation performance in this simulation case.

5. Conclusion and discussion

The IF estimation of multicomponent NLFM signals in low SNR has become a research hotspot in modern signal processing. This paper introduced the generalized S-transform to NLFM signal processing field and proposed an IF estimation algorithm of NLFM signals based on generalized S-transform, which solved the difficult problem of IF estimation of multicomponent NLFM signals in low SNR. Simulation results showed that the algorithm proposed in this paper has better performance in IF estimation than other existing algorithms, which provided a novel thought and method to IF estimation of NLFM signals in low SNR. Nonetheless, the IF estimation algorithm proposed in this paper has a large compute complexity, the future research should be focused on reducing computational effort and improving processing speed on the premise of ensuring the accuracy of parameters estimation.

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7. References


