Dynamic Schedule Strategy for Urban Rail Transit

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Abstract

The work efficiency of urban rail transit mostly depends on the train schedule strategy, as it could influence the balance of energy saving and passenger flow pressure to station site. In this paper, a new approach to train dispatching and schedule is introduced to optimize metro operation. Each station site adds up the amount of coming and exiting passengers as history data, and then, passenger flow prediction is carried out through Kalman filter. After data analyzing, the center server adjusts the train timetable dynamically using the exponent function.

Key words: Dynamic Train Schedule, Passenger Flow Prediction, Kalman Filter, Train Timetable

1. Introduction

With the rapid development of urban infrastructure, urban rail transit (also called urban mass rapid transit, MRT) has become more and more important in people’s daily life. Compared to common ground traffic, its features include but not limited:

- Large carrying capacity
- High efficiency and Fast
- Environment friendly

Based on the advantages above, MRT construction program has been a dispensable component in city infrastructure plan. Contrast to the progress of MRT signal process method and hardware facilities, train schedule algorithm which may ensure the MRT system running safely and orderly is still at a very low level. A good way to train management could enhance the whole performance of MRT without plus extra source.

However, in the current period, there is no mature theory on MRT schedule has been raised as general standard, as well as few research organizations devote their energies to this field, most research achievements are focused on communication hardware's development of MRT schedule system. Almost all MRT conduct centers still use the traditional and rigid routine to manage train’s running.
The figure above shows that, train schedule schemes mostly rely on dispatcher’s individual experience. To the general schedule hub, it is not allowed to revise the running program arbitrarily due to it is a complicated and tremendous work\(^3\).

In this context, a novel approach to dynamic schedule for MRT will be introduced. Each station site counts the number of passengers who enter and exit respectively. And then, it predicts the passenger flow on the basis of history data combined with Kalman filter. Meanwhile, the station site receives data of passenger flow from adjacent station site. Thereby, the station site could foresee whether the future passenger flow would exceed the maximum carrying capacity. Finally, stations send the outcomes to the center server, which adjusts the timetable in certain way.

In the final, some suggestions should be noted in the whole paper: The starting number of train is from 1; the starting number of time is from 0.

2. Passenger Flow Prediction

A good passenger flow prediction algorithm means a good schedule strategy program since schedule strategy counterpoise the pressure of passenger flow to station site and energy saving. Too many numbers of trains in unit time reduce passengers’ waiting time as well as consume more power. Otherwise, the situation is opposite. The following section will discuss the passenger flow prediction in the short time future.

2.1. History Statistic Data

In the real world, there are a plenty of factors that affect the total passengers in MRT, for instance: weather, season, policy etc. It is seldom possible to foresee the amount of passengers accurately in the future. All information we can acquire must be based on the history statistic data.

At the certain moment, the total numbers of passengers on train number j at station site i could be acquired from the equation\(^2\):

\[
N_{ij} = \sum_{k=1}^{t} (C_{kj} - O_{kj})
\]

where:

\(N_{ij}\): The total numbers of passengers on train number j at station i.
\(C_{kj}\): The total numbers of passengers enter into station site k and get on the train number j.
O_k: The total numbers of passengers alight from train number j and exit from station site k. Assuming that, X and Y are random variables, which represent number of passengers who enter and exit respectively in unit time.

Table 1. Passengers sample

<table>
<thead>
<tr>
<th>Time</th>
<th>t_1-t_0</th>
<th>t_2-t_1</th>
<th>t_3-t_2</th>
<th>......</th>
<th>t_n-t_{n-1}</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>x_1</td>
<td>x_2</td>
<td>x_3</td>
<td>......</td>
<td>x_{n-1}</td>
</tr>
<tr>
<td>Y</td>
<td>y_1</td>
<td>y_2</td>
<td>y_3</td>
<td>......</td>
<td>y_{n-1}</td>
</tr>
</tbody>
</table>

z_i means the amount of passengers who come in during the time t_i-t_{i-1}. As the observed data fluctuates at the same time on different day, data could be gathered in the following way: observing the passenger flow at the same time for N days (such as 30 days), then calculating the mean value at the same time. Therefore, x_i could be figured out with the equation below:

\[ x_i = \frac{\sum_{j=1}^{N} x_{ij}}{N} \]  (2)

where 
- x_{ij} represents the amount of passengers who come in on day j during the time t_i-t_{i-1}
- y_i means the amount of passengers who go out during the time t_i-t_{i-1}. Analog to x_i, the calculation method for them are the same.

In the following part, X and Y will be expressed in vector form in the rest of this paper.

2.2. Passenger flow prediction based on Kalman filter

Kalman Filter that provides a highly effective calculation method to estimate the state of the process with the minimum mean square error is consist of a set of recursive mathematical formula. It has been widely used to estimate the status of signal in the past, current even the future even without knowing the exact model character. The typical application of Kalman Filter is to predict the coordinate and speed of object from the observed data which is limited and contains noise, as well as it is an important subject on control theory and control system. It has been applied in many fields such as radar and computer vision.

In order to use the Kalman filter to estimate the internal state of a process given only a sequence of noisy observations, one must model the process in accordance with the framework of the Kalman filter. This means specifying the following matrices: F_k, the state-transition model; H_k, the observation model; Q_k, the covariance of the process noise; R_k, the covariance of the observation noise; and sometimes B_k, the control-input model, for each time-step, k, as described below.

The Kalman filter model assumes the true state at time k is evolved from the state at (k − 1) according to

\[ x_k = F_k x_{k-1} + B_k u_k + w_k \]  (3)

where
- \( F_k \) is the state transition model which is applied to the previous state \( x_{k-1} \);
- \( B_k \) is the control-input model which is applied to the control vector \( u_k \);
- \( w_k \) is the process noise which is assumed to be drawn from a zero mean multivariate normal distribution with covariance \( Q_k \).

\[ w_k \sim N(0, Q_k) \]  (4)

At time k an observation (or measurement) \( z_k \) of the true state \( x_k \) is made according to

\[ z_k = H_k x_k + v_k \]  (5)

where
- \( H_k \) is the observation model which maps the true state space into the observed space and \( v_k \) is the observation noise which is assumed to be zero mean Gaussian white noise with covariance \( R_k \).

\[ v_k \sim N(0, R_k) \]

The initial state, and the noise vectors at each step \{x_0, w_1, ..., w_k, v_1, ..., v_k\} are all assumed to be mutually independent.
The Kalman Filter can be written as a single equation; however it is most often conceptualized as two distinct phases: "Predict" and "Update".

**Predict**
- Predicted (a prior) state estimate
  \[ \hat{x}_{k|k-1} = F_k \hat{x}_{k-1|k-1} + B_k u_k \]  
  \[ (6) \]
- Predicted (a prior) estimate covariance
  \[ P_{k|k-1} = F_k P_{k-1|k-1} F_k^T + Q_k \]  
  \[ (7) \]

**Update**
- Innovation or measurement residual
  \[ \tilde{y}_k = z_k - H_k \hat{x}_{k|k-1} \]  
  \[ (8) \]
- Innovation (or residual)
  \[ S_k = H_k P_{k|k-1} H_k^T + R_k \]  
  \[ (9) \]
- Optimal Kalman gain
  \[ K_k = P_{k|k-1} H_k^T S_k^{-1} \]  
  \[ (10) \]
- Updated (a posteriori) state estimate
  \[ \hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k \tilde{y}_k \]  
  \[ (11) \]
- Updated (a posteriori) estimate covariance
  \[ P_{k|k} = (I - K_k H_k) P_{k|k-1} \]  
  \[ (12) \]

Practical implementation of the Kalman filter is often difficult due to the inability in getting a good estimate of the noise covariance matrices \( Q_k \) and \( R_k \). At present, most researchers determine the parameters rely on their experience or observed data on statistic. In this paper, each parameter is defined as follow: \( Q_k \) equals to \( D(X) \); \( R_k \) equals to \( D(Y) \). Consequently, the formula of Kalman filter is modified as the below:

\[ \begin{align*}
\hat{x}_{k|k-1} &= F_k \hat{x}_{k-1|k-1} \\
P_{k|k-1} &= F_k P_{k-1|k-1} F_k^T + D(X) \\
\tilde{y}_k &= z_k - H_k \hat{x}_{k|k-1} \\
S_k &= H_k P_{k|k-1} H_k^T + D(Y) \left( \sum_{j=1}^{k-1} I_k - O_k \right) \\
K_k &= P_{k|k-1} H_k^T S_k^{-1} \\
\hat{x}_{k|k} &= \hat{x}_{k|k-1} + K_k \tilde{y}_k \\
P_{k|k} &= (I - K_k H_k) P_{k|k-1}
\end{align*} \]  
\[ (13) \]

### 2.3. Parameters Determination

The parameters determination of Kalman Filter is the most difficult. At present, technicians always define the value of the parameters by their own experience, as well as few classical mathematical models could be applied to optimize parameters. In this paper, parameters will be figured out on the basis of statistic data feature in section 2.1.

#### 2.3.1. Transfer Matrix

Transfer matrix was considered in two aspects: the passengers’ ratio in adjacent time from statistic information and the relevance of history and observed data.

<table>
<thead>
<tr>
<th>Time</th>
<th>( t_1-t_0 )</th>
<th>( t_2-t_1 )</th>
<th>( t_3-t_2 )</th>
<th>( t_4-t_3 )</th>
<th>\ldots</th>
<th>( t_n-t_{n-1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>( x_1 )</td>
<td>( x_2 )</td>
<td>( x_3 )</td>
<td>( x_4 )</td>
<td>\ldots</td>
<td>( x_n )</td>
</tr>
<tr>
<td>O</td>
<td>( o_0 )</td>
<td>( o_2 )</td>
<td>( o_3 )</td>
<td>( o_4 )</td>
<td>\ldots</td>
<td>( o_n )</td>
</tr>
<tr>
<td>F</td>
<td>( F_1 )</td>
<td>( F_2 )</td>
<td>\ldots</td>
<td>( F_{n-2} )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
We only take random variable $X$ as example here, $Y$ could be figured out in the same way. $X$ is the history data based on statistic, and we know how to acquire it from section 2.1; $O$ represents the watching data, it could be told by the automatic ticket gates; $F$ is the variable for transfer matrix.

$$F = \begin{pmatrix} x_{i+2} \\ x_{i+1} \end{pmatrix}$$  \hfill (14)

### 2.3.2. Error Parameters

$(\delta, \sigma^2)$ are the symbol of expectation and variance for $X$ and $Y$, the mathematical expectation and variance of $X$ and $Y$ will be figured out as the parameters of Kalman Filter. According to the table 1, assuming that passenger flow data was observed in $N$ days, $M$ represents the arithmetic mean of data in $N$ days.

$$M = \frac{\sum x_i + \sum y_i}{N}$$  \hfill (15)

And then, mathematical expectation and variance could be gained

$$E\left[\begin{bmatrix} X \\ Y \end{bmatrix}\right] = \delta = \frac{\sum x_i}{N}$$  \hfill (16)

$$D\left[\begin{bmatrix} X \\ Y \end{bmatrix}\right] = \sigma^2 = \frac{\sum (x_i - E(X))^2}{N}$$  \hfill (17)

In this paper, matrix $\sigma$ donates to $Q_k$ and $R_k$, it is reasonable under experience.

### 3. Time table adjustment

#### 3.1. Adjustment Condition

Let us modify the result form of equation 14. $p_{xit}$ represents the predicted passenger flow that comes into station site during time $t$ at station $i$; $p_{yit}$ means the predicted passenger flow that goes out from station during time $t$ at station $i$.

Train arriving information could be obtained from timetable via net or other channels. During the time $s$ (future time), total number of people $N_i$ on a certain train at station $i$ could be acquired from the formula 1:

$$N_i = \sum_{k=1}^{s} (p_{aks} - p_{aks})$$  \hfill (18)

Assuming that, $C$ means the carrying capacity of one train, the conclusion shows below

$$\varepsilon = C - N_i = \begin{cases} 
\geq 0 & \text{Timetable needs adjusting} \\
< 0 & 
\end{cases}$$  \hfill (19)

#### 3.2. Adjustment Strategy

The time record on timetables assumed to be Item$_1$, Item$_2$, Item$_3$ … Item$_n$. Assigning the unit second to $\varepsilon$, $e^\varepsilon$ is chosen to be the adjustment function. If the timetable needs adjusting, the way shows below

$$i = n + (i - \varepsilon \cdot e^{i+1})$$  \hfill (20)
The modification strategy influences the item which lies at top of the timetable most strongly on the basis of experience. Additionally, $\varepsilon \cdot \varepsilon_{i+1}$ must be controlled in a rational range, such as $[0, 10]$. As the random variable $P$ is discrete, its interval may be decided by technicians. For the purpose of computation, linear interpolation is applied, so the number of passengers in any time segment could be obtained.

4. Experiment Result

All data below is generated randomly, it is just for experiment. The experiment takes station site 3 as an example.

**Table 3. Train Timetable**

<table>
<thead>
<tr>
<th>No.</th>
<th>AT1</th>
<th>AT2</th>
<th>AT3</th>
<th>AT4</th>
<th>AT5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Item1</td>
<td>1:00</td>
<td>1:20</td>
<td>1:50</td>
<td>2:10</td>
<td>2:25</td>
</tr>
<tr>
<td>Item2</td>
<td>1:30</td>
<td>1:50</td>
<td>2:20</td>
<td>2:40</td>
<td>2:55</td>
</tr>
<tr>
<td>Item3</td>
<td>1:40</td>
<td>2:00</td>
<td>2:30</td>
<td>2:50</td>
<td>3:05</td>
</tr>
<tr>
<td>Item4</td>
<td>2:00</td>
<td>2:20</td>
<td>2:50</td>
<td>3:10</td>
<td>3:25</td>
</tr>
</tbody>
</table>

*ATn=Arriving Time at station n

**Table 4. Passenger Flow Information**

<table>
<thead>
<tr>
<th>Time</th>
<th>1:00-1:20</th>
<th>1:20-1:40</th>
<th>1:40-2:00</th>
<th>2:00-2:20</th>
<th>2:20-2:40</th>
<th>2:40-3:00</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>100</td>
<td>120</td>
<td>145</td>
<td>100</td>
<td>85</td>
<td>80</td>
</tr>
<tr>
<td>O</td>
<td>112</td>
<td>125</td>
<td>168</td>
<td>83</td>
<td>65</td>
<td>50</td>
</tr>
<tr>
<td>$P_x$</td>
<td>1.2</td>
<td>0.69</td>
<td>0.69</td>
<td>0.85</td>
<td>0.94</td>
<td></td>
</tr>
<tr>
<td>$\gamma$</td>
<td>93</td>
<td>107</td>
<td>110</td>
<td>95</td>
<td>90</td>
<td>90</td>
</tr>
<tr>
<td>$P_y$</td>
<td>100</td>
<td>103</td>
<td>120</td>
<td>102</td>
<td>85</td>
<td>60</td>
</tr>
<tr>
<td>$P_{y}$</td>
<td>1.03</td>
<td>0.86</td>
<td>0.95</td>
<td>1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

From the formula 16 and 17, we know the expectation and variance of $X$ and $Y$

$$E_X[Y] = \delta = \frac{\sum_{k=1}^{n} \gamma_k}{\sum_{k=1}^{n}} = \frac{105}{97.5} = 1.075$$

(21)

$$\text{D}E_Y = \sigma^2 = \frac{\sum_{k=1}^{n}(\gamma_k - E(Y))^2}{\sum_{k=1}^{n}} = \frac{1460}{385.5} = 3.80$$

(22)

120 is given to constant $C$, which means maximum value of carrying. Take all the information into equation group 3. Through the calculation above, once the maximum people amount exceeds the value $C$, the center server could shorten the interval time of train departure to relieve the stress to station site.

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6. References