A Novel Iterative Algorithm for Time-varying Channel Estimation in OFDM systems

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Abstract

A novel iterative channel estimation algorithm for orthogonal frequency-division multiplexing (OFDM) systems in fast time-varying environment is proposed. Instead of directly estimate the original channel taps, the proposed estimation focus on the rest channel part from the original channel taps subtracted by the prior estimation results in the absence of any channel information. The initial results are obtained by time-domain least squares (LS) estimation by inserting zeros-padded training symbols between adjacent OFDM symbols. Monte Carlo simulations demonstrate the superiority of the proposed algorithm after a few iterations over the conventional methods based on complex-exponential basis expansion model (CE-BEM) or generalized complex-exponential basis expansion model (GCE-BEM).

Keywords: Channel Estimation, Basis Expansion Model (BEM), Time-varying Channel

1. Introduction

In highly mobile wireless applications, channel estimation plays a vital role in equalization, coherent detection, and demodulation in orthogonal frequency-division multiplexing (OFDM) systems. The impact of the channel on the transmitted signal must be estimated to recover the original information. Differential modulation techniques have been taken into consideration to avoid the complicated channel estimation procedure, however, such scheme leads to an inevitable penalty for 3-4dB SNR [1].

The tough challenge for the channel estimation due to high-mobility is that the number of parameters to handle is much larger than that of observable data. In light of Jakes model for time-varying channel [2], we can approximate the time-variant complex gains by the following channel models to reduce the estimation burden.

Mostofi and Cox proposed a piece-wise linear model to approximate channel time-variations in [3]. In this linear model, the slope is obtained by cyclic prefix (CP) in an iterative way or the average gains of previous and next symbol. Nevertheless, the time average of the multipath complex gains over the effective duration of each OFDM symbol is somehow contaminated by inter-carrier interference (ICI) [4]. Regard this, Hijazi et al. suggested polynomial model [5] or low-pass interpolation [6] to refine the model coefficients with the coarsely estimated ICI. An auto-regressive (AR) model is built to update the polynomial coefficients dynamically [7]. The repeated-pattern preamble is employed to collect channel information of the previous symbol for ICI pre-canceling [8]. Anyway, the performance of linear model degrades considerably with the increase of mobility and the ICI reduction effects will also be constrained by the modeling error.

Without any channel information, complex-exponential BEM (CE-BEM) [9] and generalized CE-BEM (GCE-BEM) [10] are popular due to easy generation of the orthogonal kernel matrix and fast algebra operation using fast Fourier transformation. However, an effect of Gibbs phenomenon, as well as spectral leakage, leads to significant errors at the edges of the data blocks. A new windowing and de-windowing technique was used to eliminate the high frequency component to reduce the spectral leakage for better performance [11]. Based on the knowledge of the maximum Doppler frequency, a modification of CE-BEM constraints the modeling frequency bandwidth to reduce the out-of-band noise [12]. In general, CE-BEM and GCE-BEM are preferable for fast fading channel without any prior information.

In this paper, an iterative algorithm for channel estimation is proposed to significantly improve the estimation accuracy by reducing the modeling error. Given the prior estimation results, we subtract the
original part by prior results and estimate the rest part. With the increase of iterations, the estimation result in terms of mean square errors (MSE) and the system performance in terms of bit error rate (BER) improve considerably.

The rest sections are organized as follows. Section 2 presents the OFDM system model and the conventional estimation can be found in section 3. In section 4, we propose the iterative channel estimation algorithm for better estimation performance. Simulations in section 5 prove the advantage and section 6 summaries our study.

2. OFDM System Model

In OFDM systems, the spectrum is divided to N orthogonal subcarriers. \( X(n) \) is a modulated complex-valued frequency-domain transmitted data on the \( n \)-th subcarrier. An \( N \)-point inverse fast Fourier transform (IFFT) is applied to the symbol \( X(n) \) to get time-domain signal \( x(k) \) as

\[
x(k) = \frac{1}{N} \sum_{n=0}^{N-1} X(n) \exp\left(\frac{j2\pi kn}{N}\right)
\]

(1)

After parallel-serial conversion, \( N_c \)-length CP, which is the last \( N_c \) replicas of block data \( x(k) \), is added to the beginning of each OFDM symbol to mitigate the inter-symbol interference (ISI) and preserve orthogonality of the tones. We assume that \( N_c \) is larger than the maximum delay of channel spread \( L \), hence ISI will not be considered in this paper. Besides, timing and frequency synchronization is assumed to be perfect [13].

Then, the time-domain signal is launched to the doubly selected channel and the information symbol \( y(k) \) is decoupled from the received signal by discarding the corresponding CP component, the vector form of which is given by

\[
y = Hx + w
\]

(2)

where \( y = [y(0), \ldots, y(N-1)]^T \), \( x = [x(0), \ldots, x(N-1)]^T \) and \( H \) is an \( N \times N \) circulant matrix with the \((m,n)\)-th element given by

\[
[H]_{m,n} = \begin{cases} h(m-1; m-n) & m \geq n \text{ and } m-n < L \\ h(m-1; m-n+N) & m < n \text{ and } m+n+N-n < L \\ 0 & \text{otherwise} \end{cases}
\]

where \( h(n;l) \) is the \( n \)-th sampling gain in \( l \)-th channel impulse response.

Afterward, the fast Fourier transform (FFT) is applied to \( y \), and the frequency-domain output is

\[
Y = FY = GX + Fw
\]

(3)

where \( Y \) and \( X \) are the frequency-domain output and input, respectively. \( F \) is the normalized Fourier transform matrix with the element \([F]_{m,n} = 1/\sqrt{N} \exp(-j2\pi mn/N)\) and \( G = FHF^H \) indicates the channel frequency response.

If the channel is invariant during the block transmission, i.e., \( h(n;l) = h(m;l) \) for \( 0 \leq n, m < N \), \( G \) is a dialog matrix. Otherwise, ICI is induced since \( G \) is not diagonal any more. Therefore, equalization and ICI mitigation necessitate the accurate estimation of time-varying channel.
3. Conventional GCE-BEM Estimation

To reduce the estimation burden, GCE-BEM has been introduced to model the elements of each tap due to the correlation of all the elements. The general model for the \(l\)-th tap is given by

\[
b_l = \sum_{q=0}^{Q} b_q c_q(l) + e
\]  

(4)

where \(b_q = [b_q(0), \ldots, b_q(N-1)]^T\) is the \(q\)-th basis vector with \(N\) elements, \(c_q(l)\) is the model coefficient to estimate first and \(e\) is the modeling error. As a consequence, \(N\) parameters of each tap to estimate are represented by \(Q + 1\) coefficients. In other words, the \(N\)-dimension original space is cast to \(Q + 1\) model space with vanishingly small modeling error to facilitate the estimator.

For GCE-BEM, the \(n\)-th element in \(q\)-th basis vector is given by

\[
b_q(n) = \frac{1}{\sqrt{N}} \exp\left(j2\pi \frac{q - \frac{Q}{2}}{NP} n\right)
\]

(5)

where the scale \(\sqrt{N}\) is used here for normalization and \(P\) is an positive integer larger than 1.

Substituting (4) into (3) and after some algebra, the basic estimation formula can be derived as [14]

\[
Y = D\hat{c} + \tilde{W}
\]

(6)

where

\[
D = [D_0, D_1, \ldots, D_q] \quad \{I_N, \otimes \tilde{X}\} \\
\tilde{X} = \sqrt{N} \cdot \text{diag}\{X\} F_L \\
\hat{c} = \begin{bmatrix} c_0^T, c_1^T, \ldots, c_Q^T \end{bmatrix}^T \\
D_q = F \text{diag}\{b_q\} F_H^H \\
c_q = \begin{bmatrix} c_q(0), \ldots, c_q(L-1) \end{bmatrix}^T
\]

and the term \(\tilde{W}\) includes both the channel noise and the modeling error. In the above formulas, \(I_N\) denotes \(N \times N\) entity matrix. \(\text{diag}\{x\}\) represents the a diagonal matrix with \(x\) as its diagonal. \(\otimes\) represents the Kronecker product. \((\cdot)^T\), \((\cdot)^H\), and \((\cdot)^H\) stand for conjugate, transpose, and complex conjugate transpose (Hermitian), respective. And the term \(F_L\) consists of the first \(L\) columns of \(F\).

In addition, \(K\) equidistant pulse-shaped pilot clusters with index \(p_0, p_1, \ldots, p_{K-1}\) are induced to maximize a lower bound on the average channel capacity [15]. The superscripts \((p)\) and \((d)\) are denoted as the corresponding part to the pilots and data. Then, (6) is changed into the pilots form

\[
Y^{(p)} = D^{(p)} \hat{c} + D^{(d)} \hat{c} + \tilde{W}^{(p)}
\]

(7)

Based on formula (7), least-square (LS) estimator or minimum mean square error (MMSE) estimator can be employed to obtain the BEM coefficients vectors \(\hat{c}\). In the proposed algorithm, LS estimator is used due to its easy implementation. Besides, the prior channel information is not always available.
4. Proposed Iterative Channel Estimation Algorithm

4.1. Initial Time Domain Estimation

In order to obtain the initial estimation for the proposed iterative channel estimation, the training symbols are inserted between two successive OFDM symbols in time domain with the pattern in Figure 1. The known symbols of length $L_1$ are padded by two zeros guard periods of length $L_2$. We denote the known symbols with the following guard before the $k$-th OFDM symbol as $\hat{x}_k$ of length $L_1 + L_2$ and the corresponding part at the receiver is $\hat{y}_k$.

We assume that the channel is constant during the $2L_2$ samples period since $2L_2 < N$ and $L_1$ is greater than the maximum channel delay $L$. Then, the channel impulse response just before the $k$-th OFDM symbol $\hat{h}_k$ can be calculated by

$$\hat{y}_k = \hat{x}_k \hat{h}_k + w_k \quad (8)$$

In (8), $w_k$ is AWGN and $\hat{x}_k$ is a $(L_1 + L_2) \times L$ matrix with the elements as

$$\left[ \hat{x}_k \right]_{m,n} = \begin{cases} \hat{x}_{k,m-n} & m \geq n \\ 0 & \text{otherwise} \end{cases} \quad (9)$$

Using LS estimator, we have

$$\hat{h}_k = \hat{x}_k^H \hat{y}_k \quad (10)$$

As illustrated in Figure 1, the initial channel estimation within $k$-th OFDM symbol duration (denoted by the dash line in Figure 1) can be obtained by the linear interpolation between $\hat{h}_k$ and $\hat{h}_{k+1}$.

4.2. Estimation with Prior Iteration Results

Given the $l$-th estimated tap $\hat{h}_j^{prior}$ from the prior iteration or the initial estimation for the first time, the rest of part of the channel tap can be estimated as follows

$$h_j - \hat{h}_j^{prior} = \sum_{q=0}^{Q} b_q e_q(l) + e \quad (11)$$

Substituting (11) into (6), we have

$$Y = D\hat{c} + FDF^H X^{prior} + W \quad (12)$$
where $X^{\text{prior}}$ represents the equalization results of the prior iteration and the element of $\mathbf{D}$ is given by

$$
\mathbf{D}_{m,n} = \begin{cases} 
    h_{m,n}^{\text{prior}}(m) & m \geq n \& m - n < L \\
    h_{m,n}^{\text{prior}}(m) & m < n \& m + N - n < L \\
    0 & \text{otherwise}
\end{cases}
$$

Hence, by replacing $\mathbf{Y}$ by $\mathbf{Y} - \mathbf{FDF}X^{\text{prior}}$ in (6), the BEM coefficients can be estimated in the conventional method. As a consequence, the ultimate estimated tap is

$$
\hat{h}_c = h_c^{\text{prior}} + \sum_{q=0}^{Q} b_q \hat{c}_q(l)
$$

where $\hat{c}_q(l)$ is estimated via conventional method.

### 4.3. Iterative Channel Estimation Algorithm

The estimation algorithm proceeds as follows:

**Initialization:**
- $X^{\text{prior}}$ is the vector with only pilots, and other elements are set nulls.
- $\mathbf{Y}$ is the output of FFT.
- $h_c^{\text{prior}}$ is calculated by linear interpolation of the inserted training symbols in time domain(10).

**Recursion:**
1. calculate the channel matrix $\mathbf{D}$ using the prior results $h_c^{\text{prior}}$ (13);
2. execute the conventional channel estimation of the rest part by replacing $\mathbf{Y}$ with $\mathbf{Y} - \mathbf{FDF}X^{\text{prior}}$ (6);
3. combine the prior result $h_c^{\text{prior}}$ and the estimated rest part to get the channel impulse response (14) and update $h_c^{\text{prior}}$ for the next iteration;
4. detect $X$ after removing the ICI from the pilots and update $X^{\text{prior}}$.

The complexity is proportional to the times of iterations and the iteration heavily depends on the computation complexity of the conventional method. Thus, it’s acceptable for small number of iterations.

### 4.4. Modeling Error Analysis

The modeling error of GCE-BEM depends on not only the selected basis vectors but also the objective channel taps to estimate. For concreteness, the discontinuity in the edges of each taps causes the frequency leakage since the high frequency components cannot be tracked with GCE-BEM and also the Gibbs phenomenon degrades the performance further. The proposed approach subtracts the prior estimated channel taps and estimates the rest part with GCE-BEM. The rest part, compared with the original tap, has much less high frequency components and varies more slightly. Thus, the proposed algorithm greatly improves the system performance.

### 5. Simulation

In this section, OFDM system with parameters in Table 1 is employed to evaluate the performance of conventional estimation methods and our proposed algorithm. The doubly selective channel consists of 4 uncorrelated taps, the power of which complies with negative exponential distribution. According to the Jakes’ model [2], each tap with 25 different paths in it is time-variant Rayleigh random process.
as depicted in [16]. The proposed algorithm is tested in terms of MSE and BER for different SNR and normalized Doppler frequency $f_d$.

### Table 1. OFDM system parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
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<tr>
<td>Subcarrier bandwidth</td>
<td>15kHz</td>
</tr>
<tr>
<td>FFT/IFFT points</td>
<td>256</td>
</tr>
<tr>
<td>CP length</td>
<td>8</td>
</tr>
<tr>
<td>Pilot/Data modulation</td>
<td>QPSK</td>
</tr>
<tr>
<td>Cluster width</td>
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<tr>
<td>Number of clusters</td>
<td>9</td>
</tr>
<tr>
<td>BEM dimension</td>
<td>5</td>
</tr>
</tbody>
</table>

#### Figure 2. MSE vs SNR for $f_d = 0.2$

![MSE vs SNR for $f_d = 0.2$](image)

#### Figure 3. BER vs SNR for $f_d = 0.2$

![BER vs SNR for $f_d = 0.2$](image)

Several types of BEMs are included for comparison and the involved parameters are as follows. In order to approximate the time-varying channel, we use the standard rule of thumb $Q \geq 2f_d$. In our simulation, $f_d \leq 1$ and $Q = 2$ is adequate, but we set $Q = 4$ to limit the modeling error due to the lack of dimension. For GCE-BEM and our algorithm, the overlapping factor $P = 2$.

Figure 2 and Figure 3 show the evolution of MSE and BER with the increasing SNR when $f_d = 0.2$. It’s evident that the advantage of the proposed algorithm is not obvious at low SNR but when SNR is
greater than 20dB, the algorithm outperforms the conventional estimation based on CE-BEM or GCE-BEM. In addition, the performance of the algorithm after 5 iterations is much better than that after 2 iterations. When SNR=40dB, the algorithms after 2 iterations and 5 iterations have roughly 4.6dB and 10dB gain in MSE over the conventional GCE-BEM method.

Figure 4 and Figure 5 present the MSE and BER performance for different normalized Doppler frequency $f_d$ when SNR=40dB. We test the proposed algorithm in high SNR to avoid the interference of AWGN and focus on the modeling error comparison. As $f_d$ increases, the proposed algorithm after 5 iterations has more gains in MSE over GCE-BEM and CE-BEM. Besides, the gain in BER of the proposed algorithm over GCE-BEM increases with the increasing of $f_d$.

![Figure 4. MSE vs $f_d$ for SNR=40dB](image1)

![Figure 5. BER vs $f_d$ for SNR=40dB](image2)

6. Conclusion

In this paper, we propose a novel iterative time-varying channel estimation algorithm in OFDM systems in the absence of any prior channel information. For the conventional method based on GCE-BEM and CE-BEM, the variation and the high frequency component limit the modeling accuracy. Then the proposed algorithm estimates the rest part which is obtained by subtracting the prior channel estimation from the original channel taps. Hence, the proposed algorithm outperforms the conventional estimation method based on GCE-BEM and CE-BEM.
7. Acknowledgment

This work was supported in part by the National Science and Technology Major Projects of China under Grant No. 2011ZX03001-007-03 and in part by National Natural Science Foundation of China under Grant No. 61001105.

8. References