Modeling Uncertainties Using Interval Type-2 Fuzzy Logic based on Meteorological Systems

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Abstract

Interval type-2 fuzzy logic (IT-2 FL) is the process of developing IT-2 FL inference systems. The fuzzy inference system is basically elucidating non-linear mapping of input and output using fuzzy rules. High level of uncertainty and imprecision in database brings difficulty in determining the membership function’s suitable value. This produces computational complexities of the real-world applications. In type-2 fuzzy logic systems the membership function itself is fuzzy. In this paper, we describe how IT-2 FL can be used in modeling of data applications. As a proof-of-concept application, the system was implemented based on Meteorological and Oceanographic (MetOcean) dataset. The findings show that system has better modeling performance due to the application of type-2 fuzzy sets.

Keywords: Interval type-2 fuzzy logic, Fuzzy logic, Type-2 Fuzzy Logic, MetOcean

1. Introduction

The leveraging power of human reasoning and the vast computational complexities in fuzzy systems is a major aspect that dominates the area of computing. Fuzzy logic (FL) is one of the branches in soft computing paradigm where its major contribution is to enable systems or computers to reason with uncertainty [4]. Interval type-2 fuzzy logic (IT-2 FL) helps in bringing the methods of designing the systems for satisfactory performance as well as reducing the complexities [5][8]. In the real world situation, uncertainty is everywhere. High level of uncertainty and imprecision in database brings difficulty in determining the membership function’s suitable value. In computing, there is a problem of ambiguity where Zadeh [7] introduced fuzzy sets to tackle the unambiguous manner of the systems.

The interval type-2 fuzzy inference system has been used in many areas such as: expert systems, controls and data classification [1][4][8]. However, there is less in computational modeling particularly meteorological environment. Meteorological and Oceanographic (MetOcean) is an industry that deals with much data from the ocean, meteorology, as well as oil and gas institutes. These data have some complexities in relation to computations [2][12][15]. In this paper, the design of the IT-2 fuzzy system has been developed using Mendel approach and Mamdani implications. The performance of the system has been tested using MATLAB. Furthermore, the training data used in this article were sampled from MetOcean datasets.

The structure of the paper is organized as follows. In section 2 we present the basic aspects of fuzzy logic systems. Subsequently in section 3 we provide the detail on the T-2 FL system. In section 4 we provide the retrospective discussion on the need for type-2 fuzzy logic applications in meteorological and Oceanographic environment. Finally, in section 5 we discuss and evaluate the processes taken in this research which includes the data sampling, implementation as well as the result.

2. Fuzzy Logic Systems

In this section we provide some important aspects related to fuzzy logic systems.

2.1. Fuzzy Logic Systems
Fuzzy logic is a method that attempts to mechanize and computationalize human reasoning in an environment of uncertainty or imprecision. Modeling these aspects can be carried out by employing fuzzy sets in the real-world situation [8][6]. Fuzzy logic is basically considered within the interval between false and true (0 and 1) in expressing the human reasoning. Human use propositional statements — multi-valued logic like: may be true or may be false {0, 0.5, 1} in order to reduce the ambiguity of the statement.

In general, every statement must contain hypothetical syllogism—antecedent and consequent. However, to infer (conclude) the rule base, an inference system will conceptualize these collections of if-then rules and then finally produce modus ponens {a \( \land (a \Rightarrow b) \Rightarrow b \)}. This statement means that when \( a \) and \( a \Rightarrow b \) are true, then we infer \( b \) is true.

If \( U \) is a universe of discuss and the fuzzy set \( A \) is characterized by a membership function where \( x \) is an individual element in \( A \) with interval [0, 1]. Therefore, the membership grade can be represented as:

\[
A = \{x, \mu_A(x) / x \in U\}
\]  

All the mechanization and understanding of these rules are conveyed as knowledge bases intelligently. It is noted that the natural language representation is the most significant role of fuzzy logic that serves as a cointensive reality of human knowledge expressivity [16].

**Definition 1.** Let \( x \) be a primary variable which has a range (an interval) of a membership value then the primary membership \( J_x \), can be represented as \( J_x = [\mu_{x_i}(x), \mu_{x_n}(x)] \) for all \( x' \in X \) and \( \subseteq [0, 1] \).

**Definition 2.** Let \( x \) be a variable in type-1 fuzzy set then the secondary membership function at \( x \) can be represented as \( \mu_J(x') = [J_{x_i}, J_{x_n}] \) for each \( x' \in X \) and \( \subseteq [0, 1] \). This is also called vertical slice [4][6].

### 3. Interval Fuzzy Logic System

The process of formulating and mapping variables from the input stage to the output stage using interval type-1 fuzzy logic (IT-1 FL) is called interval type-1 fuzzy inference system [4]. Due to the limited capabilities of fuzzy sets in handling the uncertainties [5], we expand the system by starting with the membership grade.

**Definition 3.** If a membership grade of all the domain is crisp set and has some interval contained in [0, 1], then the set is said to be an IT-2 Fuzzy Set.

One of the big differences between type-2 fuzzy and type-1 fuzzy set models is the usage of additional third dimension in which the occurrence of at least one fuzzy set will be an IT-2 fuzzy set in the rule base [13]. The third dimension which is related to Footprint Of Uncertainty (FOU) helps in producing better modeling of imprecision [8]. On the other hand, structurally, the presence of defuzzification block in an IT-1 FL system which results crisp value differs with the type-2 fuzzy. This means in type-2 fuzzy logic system the defuzzification block is replaced by output processing block (as in Figure 1) and consists of type-reducer and defuzzifier [6].

![Figure 1. The Structure of a Type 2 Fuzzy Logic System](image)
3.1 The Interval Type-2 Fuzzy Logic

As in Castillo and Melin [4], IT-2 FL provides an effective way for most of human reasoning which greatly supports producing the fuzzy rules. Therefore, in modeling these uncertainties of non-linear systems, we consider the Definition 4 that represents \( x \) in all points i.e. \( \forall x \in X \).

**Definition 4.** As in Mendel et al. [6, 11], \( \tilde{\Lambda} \) is said to be a type-2 fuzzy set if it has membership function \( \mu_{\tilde{\Lambda}}(x, u) \), where \( x \in X \) with \( u \in J_{x} \subseteq [0, 1] \). Then,

\[
\tilde{\Lambda} = \{(x, u), \mu_{\tilde{\Lambda}}(x, u) \mid \forall x \in X, \forall u \in J_{x} \subseteq [0, 1]\}
\]

with \( 0 \leq \mu_{\tilde{\Lambda}}(x, u) \leq 1 \). In other words,

\[
\tilde{\Lambda} = \int_{x \in X} \int_{u \in J_{x}} \mu_{\tilde{\Lambda}}(x, u) / (x, u) J \subseteq [0, 1]
\]

where \( \bigcup \) represent union of the specified values of \( x \) and \( u \).

4. MetOcean and the System Application

There have been researches on different models related to sea or particularly South East Asia Fine Grid Hindcast (SEAFINE). These researches [9][10][12] focus on prediction, simulation, and hindcast analysis. However, they have less or no reasoning capabilities in relation to human aspects. Consequently, this paper focused on modeling uncertainties using IT-2 FL system based on meteorological data.

The SEAFINE includes the following participants: BP, ChevronTexaco, ConocoPhillips, Murphy Oil, Statoil, Total, BP and Sarawak Shell Berhad [14][15]. Analytically, Ocean weather Inc. created SEAMOS (South East Asia Meteorological and Oceanographic hindcast Study) hindcast in 1992 with intention to analyze the storm influences basically in southern South China Sea [15]. SEAMOS is also known as Joint Industry Project (JIP). The precise objectives for SEAFINE are to provide: a fine grid wind and wave hindcast that constitutes 50-year continuous data, large grid resolution for waves and wind fields, and 20-50 years fine resolutions for current hindcast.

Clearly, the main reason why SEAMOS has been created is due to the need of higher spatial resolution models by SEAFINE which itself is a hindcast model. As specified by [15], the resolution is situated in the areas like the Gulf of Thailand, Java Sea and Makassar Strait. Moreover, the MetOcean data comprises: (i) wind and wave data series with grid of 6km x 6km, period of 50 years data (1956-2007) and (ii) ocean current series with grid of 12km x 12km, period of 20 years data (1981-2007).

Thus, hindcasting or hindcast models could be considered as common tools for data specification and testing numerically within the meteorological and oceanographic environment. The user can find the data only if they have such tools.

5. Discussions and Evaluation of the Method

In the earlier section, we have analyzed many illustrative papers that have been selected in the literature based on different topics focused on this work. In this part, we will elucidate and discuss the process of modeling as well as implementation of type-2 fuzzy logic system with a case study of meteorological data.

5.1. Dataset

In this study, the MetOcean datasets were used and the data was simulated using direct simulation approach (DSA) that though OSMOSIS\(^1\) software. The typical hindcast data resulted in an array format. It spanned based on: YYYYMM, DDHH, WD, WS, ETOT, TP, VMD, ETOT1, TP1, VMD1, ETOT2, TP2, VMD2 and HSIG. These mean: Year and Month, Day and Hour, Wind Direction from true North, Wind Speed, Total Wave Energy, Peak Period, Direction to, Sea energy, Peak Period of Sea, Sea Direction to, Swell energy, Peak Period of Swell, Swell Direction to, and Total Significant Wave Height respectively. We extracted the time series data for the 2005 year, ranges from 1\(^{st}\) January 2005

\(^1\)OSMOSIS (Oceanweather Software Meteorological Oceanographic Study Information System) is an ocean engineers’ analysis tool for displaying and calculating a variety of MetOcean hindcast statistics. Available at: http://www.oceanweather.com/software/
00:00 to 31st December 2005 23:00. In particular, we have considered the 104.2° longitude and 5.45° latitude of Kota Kinabalu, Malaysia.

Moreover, the total of 8760 was the size of the data. It consists of the Wind Speed (WS) as 1.08m/s to 12.88m/s and Wind Direction (WD) starts from 12.88° to 359.8°. We chose WS and WD datasets uncertainties because of their significance in practical applications. They are also the main descriptors of the parameter variabilities of meteorological systems [17]. Nonetheless, these have a great impact on the roughness and topology of MetOcean environments. The results are basically stable or unstable which are the issues of reasoning. As such, the fuzzy method needs to be applied.

5.2. Implementation Method

The presence of the high level of uncertainty and imprecision in database bring difficulty in determining membership suitable value. This complexity also involved in meteorological and oceanographic data. In order to develop and implement our method we used MATLAB(R) 2009 application. A lot of rules are used to ascertain the applicability of data. Fuzzy logic is viewed as the way of computing words rather than numbers alone. The words are captured in forms of natural language then the rules have to be applied.

In fuzzy inference systems, we constructed the rules using type-2 fuzzy logic toolbox. This toolbox is sufficient for determining the complexity of the complex systems as well as type-2 fuzzy operators [4]. We first define the rules then we applied the knowledge about MetOcean Information systems. See the rules below.

5.2.1. The Fuzzy Rule Base

Interval type-2 fuzzy rule entails IF – THEN rules that describe the system. The system model contains two inputs \( x_1 \in X_1, x_2 \in X_2 \) and one output \( y \in Y \). The rule base comprises 9 rules, \( R = 9 \) which is in form of

\[
R: \text{IF } x_1 \text{ is } K_1^i \text{ and } x_2 \text{ is } K_2^i \text{ THEN } y_1 \text{ is } C_1^i \text{ and } y_2 \text{ is } C_2^i. \]

Where the \( K_i \) and \( C_i \) are the interval type-2 fuzzy sets, with \( i = 1, 2, 3, \ldots, 9 \). This implies the consequence \( C^* = C_1 \cup C_2 \cup \ldots \cup C_9 \). See Figure 2.

![Figure 2. Fuzzy rule reasoning](image)

5.2.2. The Input Membership Functions

The primary membership function for all inputs of the IT-2 FL systems for MetOcean data inference system are Gaussian membership function with the curve depends on the parameters and of the form:
\[ \mu_{m}(x_m) = \exp\left\{ -\frac{1}{2} (x_m - \hat{x}_m)^2 \right\} \]  

(4)

where \( \sigma_{mn} \in [\sigma_{m1}, \sigma_{m2}] \) and \( m = 1, 2 \) are the non-singleton inputs for the type-2 variables, \( n = 1, 2 \) represent the bound conditions of the uncertainty of the data in relation to the standard deviation (lower and upper bound, \( \sigma_{mn} \)). The standard deviation for the uncertainty in WS was initially set to \([0.50, 1.50] \text{m/s}\) and that of WD was also initially set to \([17.83, 44.83] \text{deg.}\) See the Table 1 to Table 3.

Table 1: Mean (m) and standard deviation (\( \sigma \)) of the inputs \( x_1, x_2 \)

<table>
<thead>
<tr>
<th>Category</th>
<th>( m_{x_1} (\text{m/s}) )</th>
<th>( \sigma_{x_1} (\text{m/s}) )</th>
<th>( m_{x_2} (\text{deg}) )</th>
<th>( \sigma_{x_2} (\text{deg}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>0.6041</td>
<td>50.2434</td>
<td>17.8332</td>
</tr>
<tr>
<td>2</td>
<td>4.9710</td>
<td>0.4929</td>
<td>133.3421</td>
<td>42.6591</td>
</tr>
<tr>
<td>3</td>
<td>7.3370</td>
<td>1.3538</td>
<td>237.1235</td>
<td>44.5380</td>
</tr>
</tbody>
</table>

Table 3. Selected uncertainty interval the \( x_1 \) input

<table>
<thead>
<tr>
<th>( m_{x_1} (\text{m/s}) )</th>
<th>( m_{x_2} (\text{m/s}) )</th>
<th>( \sigma_x (\text{m/s}) )</th>
</tr>
</thead>
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<td>2.5</td>
</tr>
<tr>
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<td>2.5</td>
</tr>
<tr>
<td>3</td>
<td>7.8</td>
<td>2.5</td>
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Table 3. Selected uncertainty interval the \( x_2 \) input

<table>
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<th>( m_{x_1} (\text{deg}) )</th>
<th>( m_{x_2} (\text{deg}) )</th>
<th>( \sigma_x (\text{deg}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50</td>
<td>100</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>3</td>
<td>200</td>
<td>100</td>
</tr>
</tbody>
</table>

Figure 3. Membership for the \( x_1 \) input of the antecedent
5.2.3. The Output Membership Functions

The primary membership functions for all the antecedents have been set by Gaussian membership function then fuzzy sets consist the following equation

\[ \mu^i_m(x_m) = \exp\left\{-\frac{1}{2}\left[x_m - k_m^i / \sigma_m^i\right]^2\right\} \]  

(5)

with \( k_m^i \in [k_m^i - \sigma_m^i, k_m^i + \sigma_m^i] \) as the mean of the uncertain and the standard deviation, where the m-rules \( i = 1, 2, 3, \ldots 9 \), given \( k = 1, 2 \). The rules can be developed and understood by the knowledge representation of the antecedent of fuzzy set membership function.

5.2.4. The Consequent Membership Functions

Based on the Mamdani IT-2 inference model, the consequence of this is also a Gaussian membership function similar to the equation (3). A clear-cut manipulation was done and we received the membership functions of weather in knots (mph) for type-2 fuzzy logic systems. See Figure 5.

5.3. Result

5.3. The Fuzzy Rule Base

IT-2 fuzzy rule entails MetOcean IT-2 FL Application. Therefore, with some straightforward discretization of numerical data and applying the training samples of MetOcean datasets, we received the graph in Figure 6, the non-linear surface for MetOcean’s weather at Kinabalu. This shows that the stability of weather can be seen as a process for tuning IT-2 FL parameters. It also shows that the IT-2 FL system provides more reasonable results in weather forecasting data. These values of the elements in IT-2 FL sets are sufficient to handle uncertainty of the model knowledge.
Apparently, the fuzzy logic system here maps the fuzzy sets (of the two inputs). In comparison with Figure 1, it shows the implication of minimum and product model with the Mamdani type-2 fuzzy logic system. Notwithstanding, as in ([1][3][8][18]), this result implies that the performance of this system has improved and leads to model optimization. Hence, we recommend this approach of using IT-2 FL in optimizing and modeling the performance of fuzzy knowledge base.

6. Conclusion

In this paper, we describe how IT-2 FL can be used to produce the model value of the knowledge base. We elucidate our method using Meteorological and Oceanographic (MetOcean) dataset. The proposed method will reduce the complexity in fuzzy systems. Hence, this reduces the ambiguity of information in the knowledge base.
7. References


