The OWA-VIKOR method for multiple attributive group decision making in 2-tuple linguistic setting

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Abstract

In order to process the assessment information more exactly and make the decision result more understandable in linguistic setting, a hybrid approach integrating OWA aggregation into VIKOR for multiple attributive group decision making in 2-tuple linguistic setting is proposed. In the method, the OWA is used to aggregate the linguistic assessment information. According to the ideas of VIKOR, the positive ideal solution and negative ideal solution are got. The optimal alternative(s) is determined by the criteria that are group utility, individual regret and advantage rate linguistically. The method has exact characteristics in linguistic information processing and the result is accepted more readily. Finally, a numerical example is used to illustrate the use of the proposed method. The result shows the approach is simple, effective and easy to calculate.

Keywords: 2-tuple linguistic, VIKOR, OWA, group decision making

1. Introduction

Due to ever increasing complexity of human society, people often need to consider multiple criteria (attributes, factors, objectives) to make decisions. In many complex decision making problems, most experts tend to providing linguistic assessments rather than exact numerical values to express their opinions. The fuzzy linguistic approach is a very feasible method to handle the linguistic assessment information. The 2-tuple linguistic model [1] is a continuous model of representation of information that allows reducing the loss of information typical of other fuzzy linguistic approaches. The use of 2-tuple linguistic approach has given very good results for modeling qualitative information and it has proven to be useful in many problems, e.g., in decision making [2], tourism management [3], quality evaluation [4], political analysis [5] and risk evaluation[6].

VIKOR (VlseKriterijuska Optimizacija I Komoromisno Resenje) method is well known multiple attribute decision making (MADM) methods, which is developed by Opricovic [7]. The VIKOR method is based on an aggregation function representing “closeness to the ideal”. The VIKOR method is a compromise ranking approach for multiple criteria decision making (MCDM) problems. It determines a compromise solution, providing a maximum utility for the majority and a minimum regret for the opponent. There exists a large amount of literature involving VIKOR theory and application [8, 9, 10].

The Ordered Weighted Averaging aggregation operators, commonly known as OWA operators, are introduced by Yager [11] to provide a parameterized class of mean-type aggregation operators. It is a technique to get optimal weights of attributes based on the rank of these weighting vectors after processing aggregation. OWA operators have been widely used in computational intelligence due to their flexibility in modeling linguistically expressed aggregation instructions [12]. Also, there are some extensions[13-16], for example, OWA is extended in fuzzy linguistic approach, which is defined as LOWA [13].

In this paper a hybrid approach of OWA operator and VIKOR in 2-tuple linguistic setting is designed to incorporate the unique features from both methods to provide additional flexibility for MCDA. The remainder of the paper is organized as follows: overview of 2-tuple linguistic approach is given in Section 2; next, in Section 3 a hybrid method, integrating the OWA aggregation into the VIKOR, is constructed and explained in detail; then, Section 4 presents a numerical example to demonstrate the proposed method and, finally, some concluding remarks are furnished in Section 5.
2. 2-Tuple fuzzy linguistic approach

The 2-tuple linguistic model [1, 12] is a continuous model of representation of information that allows reducing the loss of information typical of other fuzzy linguistic approaches.

2.1 2-Tuple fuzzy linguistic term

A 2-tuple linguistic variable can be denoted as \((s_i, \alpha_i)\) where \(s_i\) denotes the central value of the \(i\)th linguistic term. \(\alpha_i\) indicates the distance to the central value of the \(i\)th linguistic term. For example, a set of five terms \(S\) could be given as follows:

\[ S = \{S_0 = VL, S_1 = L, S_2 = A, S_3 = H, S_4 = VH\} \]

It means that a linguistic term set \(S\) contains seven linguistic terms, “Very Low”, “Low”, “Average”, “High”, and “Very High”, which are denoted as \(S_0, S_1, S_2, S_3\) and \(S_4\) respectively.

2.2 Transformation of 2-tuple linguistic variable

**Definition 1** Let \(s_i \in S\) be a linguistic label. Then the function \(\theta\) used to obtain the corresponding 2-tuple linguistic information of \(s_i\) is defined as

\[ \theta : S \rightarrow S \times [-0.5, 0.5) \]

\[ \theta(s_i) = (s_i, 0), s_i \in S \]

**Definition 2** A crisp value \(\beta\) whose value belongs to interval \([0, 1]\) will be obtained after aggregating the result of evaluation using the linguistic variable set \(S\). Then the symbolic translation process is applied to translate \(\beta\) into a 2-tuple linguistic variable. The generalized translation function \((\Delta)\) can be represented as

\[ \Delta : [0, T] \rightarrow S \times [-0.5, 0.5) \]

\[ \Delta(\beta) = (S_i, \alpha_i) = \begin{cases} S_i, & i = \text{Round}(\beta) \\ \alpha_i = \beta - i, & \alpha_i \in [-0.5, 0.5) \end{cases} \]

where \(\beta \in [0, 1]\). A value \(\beta\) is translated into the closest linguistic term \(S_i\) in \(S\) with a value \(\alpha\) through the symbolic translation. The 2-tuple fuzzy linguistic approach applies the concept of symbolic translation to represent the linguistic variable using 2-tuple \((s_i, \alpha_i), s_i \in S\). The interval of value \(\alpha\) is derived from the number of linguistic terms.

**Definition 3** The 2-tuple linguistic variable can be converted into an equivalent numerical value \(\beta\) (\(\beta \in [0, 1]\)) by the following formula

\[ \Delta^{-1}(S_i, \alpha_i) = \frac{i}{g} \alpha = \beta \]

where \(\Delta^{-1}(S_i, \alpha_i)\) signifies a reverse equation for converting the 2-tuple linguistic variable into a crisp value \(\beta\). \(g\) is the number of the terms in \(S\).

2.3 Comparison of 2-tuples fuzzy linguistic variable

The comparison of linguistic information represented by 2-tuples is carried out according to an ordinary lexicographic order. If \((s_i, \alpha_i)\) and \((s_j, \alpha_j)\) are two 2-tuple linguistic variables, with each one representing a counting of information as follows:

1. when \(i > j\), \((s_i, \alpha_i)\) is better than \((s_j, \alpha_j)\);
2. If \(i = j\) and \(\alpha_i > \alpha_j\) then \((s_i, \alpha_i)\) is better than \((s_j, \alpha_j)\);
3. If \( i = j \) and \( \alpha_i < \alpha_j \) then \((s_i, \alpha_i)\) is worse than \((s_j, \alpha_j)\);

4. If \( i = j \) and \( \alpha_i = \alpha_j \) then \((s_i, \alpha_i)\) is equal to \((s_j, \alpha_j)\).

### 2.4 Operation of 2-tuple linguistic variable

**Definition 4** Suppose \( L_1 = (s_1, \alpha_1) \) and \( L_2 = (s_2, \alpha_2) \) are two 2-tuple linguistic variables. The main algebraic operations are shown as follows:

\[
L_1 \ast L_2 = (s_1, \alpha_1) \ast (s_2, \alpha_2) = (s_1 s_2, \alpha_1 + \alpha_2)
\]

where \( \ast \) and \( \ast \) symbolize the addition and multiplication operations of parameters, respectively.

**Definition 5** (negative operator). \( \text{Neg}((s_i, \alpha_i)) = \Delta (s_i, \alpha_i)) \)

**Definition 6** (Arithmetic mean). Let \( x = \{(r_1, \alpha_1), \ldots, (r_n, \alpha_n)\} \) be a 2-tuple linguistic variable set, their arithmetic mean \( \tilde{S} \) can be calculated as

\[
\tilde{S} = \Delta \left( \frac{1}{n} \sum_{i=1}^{n} \Delta^{-1}(r_i, \alpha_i) \right) = \Delta \left( \frac{1}{n} \sum_{i=1}^{n} \beta_i \right) = (r_n, \alpha_n)
\]

**Definition 7** (Weighted average operator). When \( x = \{(r_1, \alpha_1), \ldots, (r_n, \alpha_n)\} \) is a 2-tuple linguistic variable set, and \( W = \{w_1, \ldots, w_n\} \) is the weight set of linguistic terms, the 2-tuple linguistic weighted average \( \tilde{x}^* \) can be computed as

\[
\tilde{x}^* = \Delta \left( \frac{\sum_{i=1}^{n} \Delta^{-1}(r_i, \alpha_i) w_i}{\sum_{i=1}^{n} w_i} \right) = \Delta \left( \frac{\sum_{i=1}^{n} \beta_i w_j}{\sum_{i=1}^{n} w_i} \right) = (r^*, \alpha^*)
\]

**Definition 8** (Ordered weighted average operator). Let \( x = \{(r_1, \alpha_1), \ldots, (r_n, \alpha_n)\} \) be a 2-tuple linguistic variable set and \( W = \{w_1, \ldots, w_n\} \) be the linguistic 2-tuple associated weights. The 2-tuple OWA operator is computed as

\[
(r, \alpha) = \Delta \left( \sum_{i=1}^{n} h_i (r_i, \alpha_i) \right) = \Delta \left( \sum_{i=1}^{n} h_i \right) \tilde{r}, \tilde{\alpha} \in [-0.5, 0.5]
\]

Where \( c_i \) is the \( i \)th largest element in the vector \( c = (c_1, c_2, \ldots, c_l)^T \), Therefore, the OWA weight \( h_i \) is not associated with any particular value \( r_j \), rather they are associated to the ordinal position of \( c_i \).

Vector \( h = (h_1, h_2, \ldots, h_l)^T \) is computed by

\[
h_i = Q(i/l) - Q((i-1)/l), i = 1, 2, \ldots, l \text{ Where } h_i \in [0, 1] \text{ and } \sum_{i=1}^{l} h_i = 1, \text{ RIM quantifier } Q \text{ is computed by}
\]

\[
Q(r) = \begin{cases} 
0, & r < a \\
\frac{r - a}{b - a}, & a \leq r \leq b \\
1, & r > b
\end{cases}
\]

Where \( a, b, r \in [0, 1] \) In 'most', 'at least half' and 'most' the conditions, the corresponding values of \((a, b)\) are \((0.3, 0.8), (0.5, 0.5)\) and \((0.5, 1)\) respectively.

### 3. OWA-VIKOR method
The experts use the linguistic term to express their preference on the grade of importance and rating of performance with regard to attributes. Denote \( n \) alternatives under consideration as \( O_1, O_2, \ldots, O_n \) and the evaluation attribute as \( A_1, A_2, \ldots, A_q \) and the rating of each alternative \( O_i \) (\( i = 1, 2, \ldots, n \)) with respect to attribute \( A_j \) (\( j = 1, 2, \ldots, q \)) of expert \( k \) as \( b_{ij}^k \). The weight of attribute \( A_j \) (\( j = 1, 2, \ldots, q \)) of expert \( k \) is \( w_{jr}^k \).

The OWA-VIKOR method has the following steps:

Step 1: Transform the linguistic terms into the form of 2-tuple linguistic. By the transformation function \( \theta \) defined above, \( r^k_j \) and \( b_{ij}^k \) can be transformed into \( \left( r^k_j, 0 \right) \) and \( \left( b_{ij}^k, 0 \right) \), respectively.

Step 2: By the operator OWA, the aggregated grades of importance of attributes that are obtained from the experts’ linguistic assessment information can be expressed as follows:

\[
\left( r^j, \alpha_j \right) = \Delta \left( \sum_{i=1}^{n} h_{ij} \right) \quad j = 1, 2, \ldots, q
\]

Where \( m \) is the number of experts, \( \left( r^j, \alpha_j \right) \) denote the aggregated grades of importance of attribute \( A_j \).

Step 3: By operator OWA, aggregate the expert evaluation information of attributes \( \left( b_{ij}^k, \alpha_j \right) \) into the group evaluation information by

\[
\left( b_j, \alpha_j \right) = \Delta \left( \sum_{i=1}^{n} b_{ij} \right) \quad i = 1, 2, \ldots, n; \quad j = 1, 2, \ldots, q
\]

Where \( m \) is the number of experts, \( \left( b_j, \alpha_j \right) \) denote the aggregated grades of attribute \( A_j \) of alternatives \( O_i \).

Step 4: Calculate the positive ideal point \( \left( b^*, \alpha^* \right) \) and negative ideal point \( \left( b^-, \alpha^- \right) \) using the following formulas:

\[
b^*, \alpha^* = \left( \left( b^*_1, \alpha^*_1 \right), \left( b^*_2, \alpha^*_2 \right), \ldots, \left( b^*_q, \alpha^*_q \right) \right)
\]

\[
b^-, \alpha^- = \left( \left( b^-_1, \alpha^-_1 \right), \left( b^-_2, \alpha^-_2 \right), \ldots, \left( b^-_q, \alpha^-_q \right) \right)
\]

where,

\[
b^*_j = \max_{i} \left( b_{ij} \right) \quad j = 1, 2, \ldots, q
\]

\[
b^-_j = \min_{i} \left( b_{ij} \right) \quad j = 1, 2, \ldots, q
\]

Step 5: Compute the values \( S_i \) and \( R_i \) by the relations

\[
S_i = (s_i, \eta_i)
\]

\[
R_i = (r_i, \epsilon_i) = \Delta \max_{j} \left( \Delta^{-1} \left( r_{ij}, \alpha_j \right) \right) \times \Delta \left( \Delta^{-1} b_{ij} - \Delta^{-1} b_{ij} \right)
\]

\[
Q_i = (q_i, \lambda_i) = T \left( \times \frac{S_i - S^-}{S - S^-} \times (1 - v) \right) \times R_i - R^- \times R - R^- \times T \left( \times \frac{\Delta^{-1} (s_i, \eta_i) - \Delta^{-1} (s^-, \eta^-)}{\Delta^{-1} (s^+, \eta^+) - \Delta^{-1} (s^+, \eta^+)} \right)
\]

Step 6: Compute the values \( Q_i \) by the relation

\[
Q_i = (q_i, \lambda_i) = T \left( \times \frac{S_i - S^-}{S - S^-} \times (1 - v) \right) \times R_i - R^- \times R - R^- \times T \left( \times \frac{\Delta^{-1} (s_i, \eta_i) - \Delta^{-1} (s^-, \eta^-)}{\Delta^{-1} (s^+, \eta^+) - \Delta^{-1} (s^+, \eta^+)} \right)
\]
Where \( S' = (s'_{i,j}) = \min_i(s_{i,j}) \), \( S = (s_{i,j}) = \max_i(s_{i,j}) \), \( R' = (r'_{j,k}) = \min_j(r_{j,k}) \), \( R = (r_{j,k}) = \max_j(r_{j,k}) \) and \( v \) is introduced as weight of the strategy of ‘‘the majority of criteria’’ (or ‘‘the maximum group utility’’), here \( v = 0.5 \).

This compromise solution is stable within a decision making process, which could be: ‘‘voting by majority rule’’ (when \( v > 0.5 \) is needed), or ‘‘by consensus, \( v =0.5 \), or ‘‘with Veto’’ (\( v < 0.5 \)).

**Step 7:** Rank the alternatives, sorting by the values \( S_i \), \( R_i \), and \( Q_i \), in decreasing order. The results are three ranking lists. The result is a set of three ranking lists denoted as \( S[·] \), \( R[·] \) and \( Q[·] \).

**Step 8:** Propose the alternative \( O_{ij} \) corresponding to \( Q_{ij} \) (the smallest among \( Q_{ij} \) values) as compromise solution if

C1. The alternative \( O_{ij} \) has an acceptable advantage; in other words, \( Q_{ij} - Q_{ij} \geq DQ \) where \( DQ = 1/(m-1) \), and \( m \) is the number of alternatives.

C2. The alternative \( O_{ij} \) is stable within the decision making process; in other words, it is also the best ranked in \( S[·] \) or \( R[·] \). If one of the above conditions is not satisfied, then a set of compromise solutions is proposed, which consists of:

- Alternatives \( O_{ij} \) and \( O_{ij} \) where \( Q_{ij} = Q_{ij} \) if only the condition C2 is not satisfied, or
- Alternatives \( O_{ij} \), \( O_{ij} \), . . . , \( O_{ij} \) if the condition C1 is not satisfied; and \( O_{ij} \) is determined by the relation \( Q_{ij} - Q_{ij} < DQ \) for the maximum \( n \) where \( O_{ij} = Q_{ij} \) (the positions of these alternatives are in closeness).

### 4. Illustrative example

One of China's largest oil producer and supplier, as well as one of the world's major oilfield service providers and a globally reputed contractor in engineering construction. It has almost 80 scientific research institutions. It also continuously enhances technical communication with our international counterparts, jointly facing challenges in the development and utilization of oil and gas as well as environmental protection. The company is begun to implement the knowledge management. The first step is the knowledge is made explicit. However, there is lots of technology knowledge in the company. They are facing the challenge which is how to choose the most importance knowledge.

To resolve the problem of ranking the knowledge, three concerned groups of respondents, including workers, experts, and senior managers, are organized to evaluate to the technology knowledge from four attributes: •A1 is the mature;•A2 is the developing;•A3 is the core degree;•A4 is the advanced degree.

For simplicity, in the case, there are only one respondent in each group. The four possible alternatives \( O_{i}(i = 1,2,3,4) \) are to be evaluated using the linguistic term set \( S = \{V, L, A, H, VH\} \) by the three decision makers under the above four attributes, and construct the decision matrices \( B_{ij} = (b_{ij})_{4,3} \) \( (k = 1,2,3) \), and the weight decision matrices are \( R_{ij} = (r_{ij})_{4,3} \) \( (k = 1,2,3) \).

<table>
<thead>
<tr>
<th>A1</th>
<th>A2</th>
<th>A3</th>
<th>A4</th>
</tr>
</thead>
<tbody>
<tr>
<td>K1</td>
<td>A</td>
<td>H</td>
<td>VH</td>
</tr>
<tr>
<td>K 2</td>
<td>H</td>
<td>VH</td>
<td>H</td>
</tr>
<tr>
<td>K 3</td>
<td>VH</td>
<td>VH</td>
<td>VH</td>
</tr>
</tbody>
</table>
In the following, we will utilize the proposed approach in this paper getting the most desirable alternative(s):  

**Step 1.** Transforming linguistic decision matrix $B_k = (b_{ij}^{(k)})_{i,j=1,2,3}$ and $R_k = (r_{ij}^{(k)})_{i,j=1,2,3}$ into two-tuple linguistic decision matrix $B_k = (b_{ij}^{(k)},0)_{i,j=1,2,3}$ and $R_k = (r_{ij}^{(k)},0)_{i,j=1,2,3}$ respectively as follows

<table>
<thead>
<tr>
<th>K</th>
<th>O1</th>
<th>O2</th>
<th>O3</th>
<th>O4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>VH</td>
<td>VH</td>
<td>H</td>
<td>VH</td>
</tr>
<tr>
<td>2</td>
<td>H</td>
<td>VH</td>
<td>VH</td>
<td>H</td>
</tr>
<tr>
<td>3</td>
<td>VH</td>
<td>H</td>
<td>VH</td>
<td>VH</td>
</tr>
</tbody>
</table>

**Step 2.** Calculating the collective overall two-tuple linguistic weight matrix $R = (r_{ij},\alpha_{ij})_{i,j=1,2,3}$

$$R = ((VH,-0.33),(VH,0),(VH,-0.33))$$

**Step 3.** Calculating the collective overall two-tuple linguistic decision matrix $B = (b_{ij},\alpha_{ij})_{i,j=1,2,3}$

<table>
<thead>
<tr>
<th>K</th>
<th>O1</th>
<th>O2</th>
<th>O3</th>
<th>O4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>VH0</td>
<td>VH0</td>
<td>H0</td>
<td>VH0</td>
</tr>
<tr>
<td>2</td>
<td>H0</td>
<td>VH0</td>
<td>VH0</td>
<td>H0</td>
</tr>
<tr>
<td>3</td>
<td>VH0</td>
<td>VH0</td>
<td>VH0</td>
<td>VH0</td>
</tr>
</tbody>
</table>

**Step 4.** Defining the positive ideal point $(b^+,\alpha^+)$ and negative ideal point $(b^-,\alpha^-)$:

$$(b^+,\alpha^+) = ((PH,0),(VH,0),(VH,0),(VH,0))$$

$$(b^-,\alpha^-) = ((H,0),(H,-0.33),(H,-0.33),(H,0))$$

**Step 5.** We assume $\nu=0.5$ and compute the values $S_i, R_i$ and the result is shown in table 1.
Table 6. Values of $S_i, R_i$

<table>
<thead>
<tr>
<th>O1</th>
<th>O2</th>
<th>O3</th>
<th>O4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{(VL,0)}$</td>
<td>$S_{(VH,0)}$</td>
<td>$S_{(L,0.42)}$</td>
<td>$S_{(H,0.42)}$</td>
</tr>
<tr>
<td>$R_{(VL,0)}$</td>
<td>$R_{(L,0.43)}$</td>
<td>$R_{(L,-0.22)}$</td>
<td>$R_{(A,0.08)}$</td>
</tr>
</tbody>
</table>

Step 6. Compute the values $Q_i, Q_i = (L,0), Q_i = (A,0), Q_i = (VL,0.45), Q_i = (VL,0.33)$.

Step 7 and step 8. Ranking all the alternatives $O_i (i = 1,2,3,4)$: $O_2 > O_1 > O_3 > O_4$, and thus the most desirable alternative is $O_2$.

5. Conclusions

In this paper, a hybrid approach is designed to integrate OWA operators into the VIKOR in two-tuple linguistic setting analysis to achieve diverse linguistic information aggregations for multiple criteria decision analysis. The designed method incorporates the unique features from the three methods for MCDA. It has the advantages that include avoiding loss and distortion of experts’ assessment information and obtaining the computation results as linguistic labels of 2-tuple linguistic model, flexibility in modeling linguistically expressed aggregation instructions of OWA and providing a maximum utility for the majority and a minimum regret for the opponent of VIKOR. Finally, a numerical example is given to verify the developed procedure and to demonstrate its effectiveness.

6. Acknowledgment

The research is supported by the National Natural Science Foundation of China under Grant No. 71101153 and No. 90924020 and the Research Funds Provided to New Recruits of China University of Petroleum-Beijing (QD-2010-06).

7. References


