A Detection and Parameter Estimation Method of Multicomponent LFM Signals Based on Reassigned S-Hough Transform

Dianwei Wang, Jiulun Fan, Yonghua Li

1School of Communication and Information Engineering, Xi’an University of Posts and Telecommunications, Xi’an, 710121, China, wangdianwei@126.com
2State Key Laboratory of Astronautic Dynamics, China Xi’an Satellite Control Center, Xi’an, 710043, China, lizenet@sohu.com

Abstract

LFM signals are widely used in the fields of radar and communication etc., which make signals detection and parameters estimation a very important technology in multicomponent LFM signals. To solve the problems of error detection caused by the cross-terms in WVD-Hough methods and other modified WVD distribution methods, and low parameter estimation precision due to poor time-frequency concentration in S-Hough transform, this paper proposed a detection and parameter estimation algorithm of multicomponent LFM signals based on reassigned generalized S-transform and Hough transform. Firstly calculate the time-frequency distribution of multicomponent LFM signals by generalized S-transform. Secondly reassign the S-transform time-frequency distribution to improve the time-frequency concentration property. Finally implement signal detection and parameter estimation by Hough transform to the reassigned S-transform TFD. The simulation results show that this algorithm has good performance in detecting multicomponent LFM signals and estimating the parameters in low SNR case.

Keywords: Detection And Parameter Estimation, Multicomponent LFM Signals, Reassigned S-Hough Transform.

1. Introduction

Linear Frequency Modulation (LFM) signals are widely used in the fields of radar and communication etc., which make signals detection and parameters estimation a very important technology in multicomponent LFM signals. To solve the problems of error detection caused by the cross-terms in WVD-Hough methods and other modified WVD distribution methods, and low parameter estimation precision due to poor time-frequency concentration in S-Hough transform, this paper proposed a detection and parameter estimation algorithm of multicomponent LFM signals based on reassigned generalized S-transform and Hough transform. Firstly calculate the time-frequency distribution of multicomponent LFM signals by generalized S-transform. Secondly reassign the S-transform time-frequency distribution to improve the time-frequency concentration property. Finally implement signal detection and parameter estimation by Hough transform to the reassigned S-transform TFD. The simulation results show that this algorithm has good performance in detecting multicomponent LFM signals and estimating the parameters in low SNR case.
simulation results show that this method has good performance in signal detection and parameter estimation to the multicomponent LFM signals, even in low SNR case.

2. Background

2.1. LFM signal

Assuming \( z(t) \) is a single-component analytic LFM signal and its express as

\[
z(t) = A(t) \exp[i\phi(t)] = A(t) \exp[j(2\pi f_c t + 0.5\pi k t^2)]
\]

where \( f_c \) is the central frequency, \( k \) is the modulation slope, \( A(t) \) is the instantaneous amplitude, \( \phi(t) \) is instantaneous phase and \( \phi(t) \in [0, 2\pi] \). When \( z(t) \) is local homochromy and meet the condition

\[
\frac{d\phi(t)}{dt} \leq \frac{1}{A(t)} \frac{dA(t)}{dt}
\]

then its instantaneous frequency (IF) can be defined as

\[
IF(t) = \frac{d\phi(t)}{dt} = 2\pi(f_c + kt)
\]

It is shown in equation (3) that the IF of LFM signal is a linear function of time variable and it expresses as a straight line on the TF plane, the processes of signal detection and parameters estimation of LFM signal can be translated into slope and intercept determine of the IF line on TF plane [6].

2.2. Theory of S-transform and its reassigned time-frequency distribution

2.2.1. Theory of S-transform

By combining the STFT and wavelet, S-transform of a time series derived by Stockwell, et al. in [10] is defined as:

\[
S(\tau, f) = \int_{-\infty}^{\infty} h(t) \sqrt{2\pi} e^{-j(f-\tau)^2} e^{-j2\pi f\tau} dt
\]

where \( S(\tau, f) \) is S-transform of \( h(t) \), \( f \) is frequency, \( \tau \) is the center of time window. From equation 4 we can know that the S-transform can be treated as STFT with adaptive Gaussian window function or a continuous wavelet (CWT) with special mother wavelet function. Whereas the width of window function of S-transform is changing with frequency \( f \) therefore it has variable resolution; at the same time the basic wavelet of S-transform do not have to meet admissibility condition, which is different to the CWT. The S-transform is a new linear time-frequency transform and has superiority characteristic of free from cross-term, so it can be applied to the analysis and processing of nonstationary signals. The window function of S-transform should meet normalized condition as

\[
\int_{-\infty}^{\infty} |h(t)|^2 dt = 1
\]

therefore S-transform and Fourier transform (FT) have relations as
\[ S(f) = \int_{-\infty}^{\infty} h(t) e^{-2\pi if \tau} d\tau = H(f) \]  

where \( H(f) \) is the FT of \( h(t) \). It is clear from equations 6 that the S-transform is nondestructive reversible which ensure it has no information loss in the interconversion between time domain and frequency domain. In consideration of the relation between S-transform and FT, the S-transform and its invert transform can be calculated by Fast Fourier transform (FFT), which enhance the calculation speed greatly and make S-transform has more extensive application prospect.

To improve the S-transform by introduced two adjust parameters and reconstruct the Gaussian window function, the generalized S-transform is formulated [12]. This allows the transform adaptively verifies the window functions base on the distribution features of frequency so it is more effective in practice. The generalized S-transform is defined as:

\[ GS(\tau, f) = \int_{-\infty}^{\infty} h(t) \left[ \frac{\lambda}{\sqrt{2\pi}} e^{-\frac{\lambda}{2}(t - \tau)^2} \right] e^{-2\pi if \tau} d\tau, \lambda, p \in [1/2, 3/2] \]  

where \( \lambda \) and \( p \) are adjustment parameters. If \( p \) is defined, the window width of the generalized S-transform broadened when \( \lambda \) increasing. In order to get high time resolution, narrower window function is chosen but the frequency resolution will be decreased at the same time due to the Heisenberg uncertainty. If \( \lambda = p = 1 \) it converts to the S-transform. So the generalized S-transform adapts to the TF analysis and filter to non-stationary signals effectively by choosing suitable parameters of \( \lambda \) and \( p \).

For a multicomponent LFM signal \( u(t) \) have \( n \) components as \( u_1(t), u_2(t), \cdots, u_n(t) \) and its expression as

\[ u(t) = \sum_{i=1}^{n} u_i(t) = \sum_{i=1}^{n} A_i(t) \exp(j\phi_i(t)) \]  

where \( A_i(t) \) and \( \phi_i(t) \) are amplitude and instantaneous phase of \( u_i(t) \) respectively. According to the definition and linear characteristic of generalized S-transform we can get the generalized S-transform TFD of \( u(t) \) as:

\[ S_u(\tau, f) = \int_{-\infty}^{\infty} u(t) w(t - \tau, f) e^{-2\pi if \tau} d\tau = \int_{-\infty}^{\infty} \left[ \sum_{i=1}^{n} u_i(t) \right] w(t - \tau, f) e^{-2\pi if \tau} d\tau 
= \int_{-\infty}^{\infty} u_1(t) w(t - \tau, f) e^{-2\pi if \tau} d\tau + \int_{-\infty}^{\infty} u_2(t) w(t - \tau, f) e^{-2\pi if \tau} d\tau + \cdots = \sum_{i=1}^{n} S_u_i(\tau, f) \]  

where \( S_u_i(\tau, f) \) is the S-transform of \( u_i(t) \). Figure 1 and figure 2 are the TFD of WVD and S-transform of a three-component LFM signal.
Figure 1. The WVD of a LFM signal.

Figure 2. The S-transform of a LFM signal.

Figure 1 shows that to a multicomponent LFM signals, the TFD of WVD has the best TF concentration, but it disturbed by the cross-term so seriously to reduce the readability. Figure 2 indicates that the S-transform is free from cross-term and has good TF concentration at the same time, which provides it unique advantages in multicomponent LFM signals detection and parameters estimation. It can be seen from figure 1 and figure 2 that the TF concentration of S-transform is lower than the WVD’s and the S-transform TFD of each LFM signal component is not a single line, which could cause large detection error if use Hough transform to detection lines on the S-transform TFD directly.

2.2.2. Reassigned TFD of S-transform [14]

It is indicated in figure 2 that the time-frequency concentration of S-transform is not very good, which could cause low precision in signal detection and parameter estimation when we use Hough transform to the S-transform TFD. To solve this problem, the reassigned TFD is adopted in this paper. The reassigned TFD is the mapping processing to each TFD point \((t, f)\) to their center of gravity \((\hat{t}, \hat{f})\) of signal energy.

\[
\hat{t}(t, f; x) = t - \frac{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S_s(t-s, f-\xi) \xi S_s(s, \xi) ds d\xi}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S_s(t-s, f-\xi) S_s(s, \xi) ds d\xi}
\]  

\[
\hat{f}(t, f; x) = f - \frac{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S_s(t-s, f-\xi) \xi S_s(s, \xi) ds d\xi}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S_s(t-s, f-\xi) S_s(s, \xi) ds d\xi}
\]

then the value of any point \((\hat{t}, \hat{f})\) of the reassigned TFD is the summation of TF value of all the points reassigned to this point and its expression as

\[
RS_s(\hat{t}, \hat{f}; x) = \int \int S_s(t, f; x) \delta(t - \hat{t}) \delta(f - \hat{f}) dt df
\]

where \(\delta(t)\) is the impulse function. Though the reassigned time-frequency process, the energy of signal on the TF plane can be redistributed and the accuracy of positioning of effective signal components can be improved. The reassigned TFD can use both the amplitude information and the phase information of the S-transform, which is the most valuable property of it.
In the practical calculation process, in order to reduce the complexity of the calculation, the equation (9) and (10) can be replaced by the follow equations:

\[
\hat{t}(x;t, f) = t - R \left\{ \frac{S_x(t, f; \mathbf{T}) S_x'(t, f; \mathbf{h})}{|S_x(t, f; \mathbf{h})|^2} \right\} \\
\hat{f}(x;t, f) = f + \text{Im} \left\{ \frac{S_x(t, f; \mathbf{D}) S_x'(t, f; \mathbf{h})}{|S_x(t, f; \mathbf{h})|^2} \right\}
\]

(12) (13)

where \( T_x(t) = t \times g(t) \), \( D_x(t) = \frac{d}{dt} h(t) \). Through reassigned process to the S-transform of figure 2, the reassigned TFD is shown as figure 3.

![Figure 3. The Reassigned S-transform TFD](image)

The reassigned S-transform TFD has better time-frequency concentration than S-transform, not only has properties information of signals but also depicts the TF regions of clusters of energy of the signals [10]. For this reason this paper reassign the S-transform TFD of multicomponent LFM signals and the result is shown as figure 3. It can be seen from the comparison of figure 1 and figure 3 that to a multicomponent LFM signal, its WVD on the TF plane is composed of fine lines of effective components (auto-term) and the cross-term; the reassigned S-transform is well identical with the fit lines of auto-term of WVD, which indicate that the reassigned S-transform of multicomponent LFM signals can serve as effective characterization of TFD of this type of signals, because of that, the detection and parameters estimation of multicomponent LFM signals can be accomplished by execute Hough transform to the reassigned S-transform.

2.3. Theory of Hough transform

The Hough transform is a straight lines detection method in transform domain and its fundamental is the duality of points and lines. The Hough transform maps straight lines in image space to a point on the Hough plane, which transform the straight lines detection processes in the image space to the points estimation operations in the parameter space [15]. The basic theory of straight lines detection with Hough transform as follows:

To a point \((x_0, y_0)\) in the image space, the normalized parameters equation of Hough transform is defined as
\[ \rho = x \cos \theta + y \sin \theta \]  \hspace{1cm} (14)

then point \((x_0, y_0)\) can be transformed to a line as \(\rho = x_0 \cos \theta + y_0 \sin \theta\) in the parameter space, then \(n\) points on the same line correspond with \(n\) lines in the parameter space of \((\rho, \theta)\). It can be known form equation (14) that all these \(n\) lines pass through the same point \((\rho_0, \theta_0)\), the corresponding line in the image space can be determined by finding the point in the parameter space. After the extraction of Reassigned S-transform of multicomponent LFM signals, we can get the image with ridge lines of each LFM signal component, and then the detection and parameters estimation processes of multicomponent LFM signals can be transformed to the searching and coordinate determination operations of local maximum value points in the Hough transform parameter space. Refer to [6] we compare the local maximum value points of Hough transform of reassigned S-transform on the TF plane and the amount of effective local maximum value points correspond to the number of LFM component in the signals. According to the coordinate of the \(i\) th local maximum value point \((\rho_i, \theta_i)\), the initial frequency \(f_0\) and modulation slop \(k_i\) of \(i\) th LFM signal component can be deduced from the equation

\[ f_0 = \rho_i / \sin \theta_i, \quad k_i = -1 / \tan \theta_i \] \hspace{1cm} (15)

3. The algorithm flow

Suppose a \(N\) component LFM signal, the main steps of LFM signal detection algorithm based on Reassigned S-transform and Hough transform proposed in this paper as:

**Step 1** Get the TFD \(S_u(t, f)\) of multicomponent LFM signal by S-transform.

**Step 2** Reassign the S-transform TFD and get the reassigned S-transform TFD.

**Step 3** Detect ridge lines \(L_u(k, f_u)\) on the reassigned S-transform TFD by Hough transform then getting points Hough transform \(P_u(\rho, \theta)\) in the parameter space.

**Step 4** Count the amount of \(P_u(\rho, \theta)\) which corresponding to the number of LFM component, then the initial frequency \(f_0\) and modulation slop of each LFM component can be deduced from the coordinate \(\rho\) and \(\theta\) of each point \(P_u(\rho, \theta)\) by equation (15).

4. Simulation results and analysis

To illustrate the validity of the method proposed in this paper, a set of 3-component LFM signal is designed with the duration time is 0.5s and the number of sampling points is 512. The initial normalized frequencies of each component are 0.1, 0.2 and 0.45 and the modulation slops are 100, 120 and -120 respectively. The relative amplitudes of each component are equal. In order to verify the performance of noise immunity of this method, a zero-mean Gaussian white noise is added to the simulation signal and the signal to noise ratio (SNR) is set to -5dB by a suitable variance is selected to the Gaussian white noise. To the simulation three-component LFM signal, the result of comparative analysis of this method with the WHHT and S-Hough is shown in figure 4.
**Figure 4.** Detection and parameter estimation results of a 3-component LFM signal. (a) Signal detection result of Wigner-Hough transform. (b) Parameters estimation result of Wigner-Hough transform. (c) Signal detection result of S-Hough transform. (d) Parameters estimation result of S-Hough transform. (e) The reassigned S-Hough transform. (f) The parameters estimation result of Reassigned S-transform and Hough transform.
5. Conclusions and suggestion

Aiming at the problems of signal detection and parameters estimation of multicomponent LFM signals, this paper proposed a detection method based on reassigned S-transform and Hough transform, which solve the detection performance decreasing problem of WHT method to multicomponent LFM signals caused by interference of cross-term and low parameters estimation precision of S-Hough method owing to poor TF concentration of S-transform. Simulation results show that this method has very good performance of signal detection of multicomponent LFM signals in low SNR, which provide a novel way of thought and method to signals detection and parameters estimation. Future studies would be needed to add appropriately processes of denosing so that the signals detection performance in low SNR case could be improved greatly.

6. Acknowledgements

This work is supported by the 2011 Special Research Programs of the Education Department of Shaanxi Province (11JK0994).

7. References


A Detection and Parameter Estimation Method of Multicomponent LFM Signals Based on Reassigned S-Hough Transform
Dianwei Wang, Jiulun Fan, Yonghua Li
