Model for Evaluating the Security of Wireless Sensor Network in Interval-Valued Intuitionistic Fuzzy Environment

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Abstract
In this paper, we investigate the multiple attribute decision making (MADM) problems for evaluating the security of wireless sensor networks with interval-valued intuitionistic fuzzy information. We utilize the interval-valued intuitionistic fuzzy hybrid geometric (IVIFHG) operator to aggregate the interval-valued intuitionistic fuzzy numbers corresponding to each alternative and derive the overall value of the alternatives, then rank the alternatives and select the most desirable one(s) according to the score function and accuracy function. Finally an illustrative example has been given to show the developed approach and demonstrate its practicality and effectiveness.

Keywords: Multiple Attribute Decision Making (MADM), Interval-Valued Intuitionistic Fuzzy Numbers, Interval-Valued Intuitionistic Fuzzy Hybrid Geometric (IVIFHG) Operator, Wireless Sensor Network (WSN)

1. Introduction

Wireless sensor networks (WSN) is a wireless self-organized network, which is composed by numerous sensors deployed densely and randomly in a geographical scenario, and is used to sense, collect and process sensed information collaboratively. Research on WSN is very significant in promoting the application of information technology in national defense, meeting the requirements of some special application field, boosting the transfer of the core technology achievements and finally popularizing WSN to social life as a new point for economic growth. As the application environment is enlarging, more and more security requirements are proposed. But we can’t solve all these problems only with the symmetric-key technology. The identity-based encryption (IBE) give use a new method to deal with such security problems. A new security solution in wireless sensor network has been proposed, with the technology of identity-based cryptography. New encryption, signature and broadcast encryption schemes has been proposed, after analyzing the security threaten in wireless sensor network, concluding the security requirements and other special requirements.

The problem of evaluating security of wireless sensor network (WSN) with interval-valued intuitionistic fuzzy information is the multiple attribute decision making (MADM) [1-16]. In this paper, we investigate the multiple attribute decision making (MADM) problems for evaluating the security of wireless sensor networks with interval-valued intuitionistic fuzzy information. The remainder of this paper is set out as follows. In the next section, we introduce some basic concepts related to interval-valued intuitionistic fuzzy variables. In Section 3 we introduce the multiple attribute decision making (MADM) problem deal with appraisal model of security of wireless sensor network (WSN) with interval-valued intuitionistic fuzzy information, in which the information about attribute weights is completely known, and the attribute values take the form of interval-valued intuitionistic fuzzy information. Then, we utilize the interval-valued intuitionistic fuzzy hybrid geometric (IVIFHG) operator to aggregate the intuitionistic fuzzy information corresponding to each alternative and get the overall value of the alternatives, then rank the alternatives and select the most desirable one(s). In Section 4, an illustrative example is pointed out. In Section 5 we conclude the paper and give some remarks.
2. Interval-Valued Intuitionistic Fuzzy Sets

Atanassov and Gargov[17] further introduced the interval-valued intuitionistic fuzzy set (IVIFS), which is a generalization of the IFS. The fundamental characteristic of the IVIFS is that the values of its membership function and non-membership function are intervals rather than exact numbers.

**Definition 1.** Let X be a universe of discourse, An IVIFS \( A \) over \( X \) is an object having the form [4-5]:

\[
\tilde{A} = \{ (x, \bar{\mu}_A(x), \bar{\nu}_A(x)) | x \in X \}
\]

(1)

Where \( \bar{\mu}_A(x) \subset [0,1] \) and \( \bar{\nu}_A(x) \subset [0,1] \) are interval numbers, and \( 0 \leq \sup(\bar{\mu}_A(x)) + \sup(\bar{\nu}_A(x)) \leq 1, \forall x \in X. \)

For convenience, let \( \bar{\mu}_A(x) = [a, b], \bar{\nu}_A(x) = [c, d] \), so \( \tilde{A} = ([a, b], [c, d]) \)

**Definition 2.** Let \( \tilde{a} = ([a, b], [c, d]) \) be an interval-valued intuitionistic fuzzy number, a score function \( S \) of an interval-valued intuitionistic fuzzy value can be represented as follows [18]:

\[
S(\tilde{a}) = \frac{a - c + b - d}{2}, \quad S(\tilde{a}) \in [-1,1].
\]

(2)

**Definition 3.** Let \( \tilde{a} = ([a, b], [c, d]) \) be an interval-valued intuitionistic fuzzy number, a accuracy function \( H \) of an interval-valued intuitionistic fuzzy value can be represented as follows [18]:

\[
H(\tilde{a}) = \frac{a + b + c + d}{2}, \quad H(\tilde{a}) \in [0,1].
\]

(3)

to evaluate the degree of accuracy of the interval-valued intuitionistic fuzzy value \( \tilde{a} = ([a, b], [c, d]) \), where \( H(\tilde{a}) \in [0,1] \). The larger the value of \( H(\tilde{a}) \), the more the degree of accuracy of the interval-valued intuitionistic fuzzy value \( \tilde{a} \).

As presented above, the score function \( S \) and the accuracy function \( H \) are, respectively, defined as the difference and the sum of the membership function \( \bar{\mu}_A(x) \) and the non-membership function \( \bar{\nu}_A(x) \). Xu[18] showed that the relation between the score function \( S \) and the accuracy function \( H \) is similar to the relation between mean and variance in statistics. Based on the score function \( S \) and the accuracy function \( H \), in the following, Xu[18] give an order relation between two interval-valued intuitionistic fuzzy values, which is defined as follows:

**Definition 4.** Let \( \tilde{a}_1 = ([a_1, b_1], [c_1, d_1]) \) and \( \tilde{a}_2 = ([a_2, b_2], [c_2, d_2]) \) be two interval-valued intuitionistic fuzzy values, \( s(\tilde{a}_1) = \frac{a_1 - c_1 + b_1 - d_1}{2} \) and \( s(\tilde{a}_2) = \frac{a_2 - c_2 + b_2 - d_2}{2} \) be the scores of \( \tilde{a} \) and \( \tilde{b} \), respectively, and let \( H(\tilde{a}_1) = \frac{a_1 + c_1 + b_1 + d_1}{2} \) and \( H(\tilde{a}_2) = \frac{a_2 + c_2 + b_2 + d_2}{2} \) be the accuracy degrees of \( \tilde{a} \) and \( \tilde{b} \), respectively, then if \( S(\tilde{a}) < S(\tilde{b}) \), then \( \tilde{a} \) is smaller than \( \tilde{b} \), denoted by \( \tilde{a} < \tilde{b} \); if \( S(\tilde{a}) = S(\tilde{b}) \), then \( \tilde{a} \) and \( \tilde{b} \) represent the same information, denoted by \( \tilde{a} = \tilde{b} \); (2) if \( H(\tilde{a}) < H(\tilde{b}) \), then \( \tilde{a} \) is smaller than \( \tilde{b} \), denoted by \( \tilde{a} < \tilde{b} \).
Definition 5. Let $\bar{\alpha} = \left[\left[\begin{array}{c} a_j^1 \\ b_j^1 \\ c_j^1 \\ d_j^1 \end{array}\right], \ldots, \left[\begin{array}{c} a_j^n \\ b_j^n \\ c_j^n \\ d_j^n \end{array}\right]\right]$ be a collection of interval-valued intuitionistic fuzzy values, and

$$IVIFHG_{\omega,w}(\bar{\alpha}, \bar{\alpha}_2, \ldots, \bar{\alpha}_n) = \prod_{j=1}^{n} \left(\tilde{\alpha}_j(\omega_j)\right)^{w_j}$$

(4)

where $\omega = (\omega_1, \omega_2, \ldots, \omega_n)$ is the associated weighting vector, with $\omega_j \in [0,1]$ such that $\sum_{j=1}^{n} \omega_j = 1$, and $\tilde{\alpha}_j(\omega_j)$ is the j-th largest element of the interval-valued intuitionistic fuzzy weighted argument $\bar{\alpha}_j(\omega_j) = \left[\left[\begin{array}{c} a_j(\omega_j) \\ b_j(\omega_j) \\ c_j(\omega_j) \\ d_j(\omega_j) \end{array}\right], \ldots, \left[\begin{array}{c} a_j(\omega_j) \\ b_j(\omega_j) \\ c_j(\omega_j) \\ d_j(\omega_j) \end{array}\right]\right]$, with $\omega_j \in [0,1]$, $\sum_{j=1}^{\infty} \omega_j = 1$, and $n$ is the balancing coefficient, then the function IVIFHG is called the interval-valued intuitionistic fuzzy hybrid geometric (IVIFHG) operator of dimension $n$[19].


Consider a multiple attribute decision making problems to deal with security assessment of Wireless Sensor Network (WSN) with interval-valued intuitionistic fuzzy information: let $A = \{A_1, A_2, \ldots, A_m\}$ be a discrete set of alternatives, and $G = \{G_1, G_2, \ldots, G_n\}$ be the set of attributes, $\omega = (\omega_1, \omega_2, \ldots, \omega_n)$ is the exponential weighting vector of the attributes $G_j$ ($j = 1, 2, \ldots, n$), where $\omega_j \in [0,1]$ such that $\sum_{j=1}^{n} \omega_j = 1$. Suppose that

$$\tilde{R} = \left[\left[\begin{array}{c} b_{ij}^1 \\ \cdots \\ b_{ij}^n \end{array}\right], \ldots, \left[\begin{array}{c} b_{ij}^1 \\ \cdots \\ b_{ij}^n \end{array}\right]\right]_{j=1}^{n} \in [0,1]$$

is the interval-valued intuitionistic fuzzy decision matrix, where $\left[\begin{array}{c} a_{ij}^1 \\ b_{ij}^1 \end{array}\right]$ indicates the degree that the alternative $A_i$ satisfies the attribute $G_j$ given by the decision maker, $\left[\begin{array}{c} c_{ij}^1 \\ d_{ij}^1 \end{array}\right]$ indicates the degree that the alternative $A_i$ doesn’t satisfy the attribute $G_j$ given by the decision maker, $a_{ij}, b_{ij} \in [0,1], c_{ij}, d_{ij} \in [0,1], b_{ij} + d_{ij} \leq 1$, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$.

In the following, we apply the interval-valued intuitionistic fuzzy hybrid geometric (IVIFHG) operator to MADM for for evaluating the computer network security with interval-valued intuitionistic fuzzy information.

Step 1. Utilize the decision information given in matrix $\tilde{R}$, and the interval-valued intuitionistic fuzzy hybrid geometric (IVIFHG) operator which has associated weighting vector $w = (w_1, w_2, \ldots, w_n)^T$

$$\tilde{r}_i = \left[\left[\begin{array}{c} a_i \\ b_i \end{array}\right], \left[\begin{array}{c} c_i \\ d_i \end{array}\right]\right] = IVIFHG_n(\tilde{r}_1, \tilde{r}_2, \ldots, \tilde{r}_n), i = 1, 2, \ldots, m, j = 1, 2, \ldots, n$$

(5)

to derive the overall preference values $\tilde{r}_i (i = 1, 2, \ldots, m)$ of the alternative $A_i$. 
Step 2. Calculate the scores \( S(\vec{r}_i) (i=1,2,\cdots,m) \) of the collective overall interval-valued intuitionistic fuzzy preference values \( \vec{r}_i (i=1,2,\cdots,m) \) to rank all the alternatives \( A_i (i=1,2,\cdots,m) \) and then to select the best one(s) (if there is no difference between two scores \( S(\vec{r}_i) \) and \( S(\vec{r}_j) \), then we need to calculate the accuracy degrees \( H(\vec{r}_i) \) and \( H(\vec{r}_j) \) of the overall interval-valued intuitionistic fuzzy preference values \( \vec{r}_i \) and \( \vec{r}_j \), respectively, and then rank the alternatives \( A_i \) and \( A_j \) in accordance with the accuracy degrees \( H(\vec{r}_i) \) and \( H(\vec{r}_j) \).

Step 3. Rank all the alternatives \( A_i (i=1,2,\cdots,m) \) and select the best one(s) in accordance with \( S(\vec{r}_i) \) and \( H(\vec{r}_i) (i=1,2,\cdots,m) \).

4. Illustrative Example

This section presents a numerical example to illustrate the method proposed in this paper. Suppose a company plans to evaluate the security of Wireless Sensor Network (WSN). There is a panel with four possible computer network systems \( A_i (i=1,2,3,4,5) \) to select. The company selects five attributes to evaluate the five possible computer network systems:

- G_1 is the tactics;
- G_2 is the technology;
- G_3 is the economy;
- G_4 is the logistics and strategy.

The five possible alternatives \( A_i (i=1,2,3,4,5) \) are to be evaluated by using the interval-valued intuitionistic fuzzy numbers by the decision makers under the above four attributes (whose weighting vector \( \omega = (0.10, 0.40, 0.30, 0.20)^T \)), and construct the decision matrix as listed in the following matrix \( \tilde{R} = (\tilde{r}_{ij})_{5 \times 4} \).

\[
\tilde{R} = \begin{bmatrix}
([0.4,0.5],[0.3,0.4]) & ([0.4,0.6],[0.2,0.4]) \\
([0.6,0.7],[0.2,0.3]) & ([0.6,0.7],[0.2,0.3]) \\
([0.3,0.6],[0.3,0.4]) & ([0.5,0.6],[0.3,0.4]) \\
([0.7,0.8],[0.1,0.2]) & ([0.6,0.7],[0.1,0.3]) \\
([0.3,0.4],[0.2,0.3]) & ([0.3,0.5],[0.1,0.3]) \\
([0.1,0.3],[0.5,0.6]) & ([0.3,0.4],[0.3,0.5]) \\
([0.4,0.7],[0.1,0.2]) & ([0.5,0.6],[0.1,0.3]) \\
([0.5,0.6],[0.1,0.3]) & ([0.4,0.5],[0.2,0.4]) \\
([0.3,0.4],[0.1,0.2]) & ([0.3,0.7],[0.1,0.2]) \\
([0.2,0.5],[0.4,0.5]) & ([0.3,0.4],[0.5,0.6]) \\
\end{bmatrix}
\]

In the following, we apply the interval-valued intuitionistic fuzzy hybrid geometric (IVIFHG) operator to MADM for evaluating the computer network security with interval-valued intuitionistic fuzzy information.

Step 1. Utilize the decision information given in matrix \( \tilde{R} \), and \( \tilde{a}_{ij} = a_{ij}^{max} \), we get:
Then by score functions, we can get:

\[
\begin{align*}
\hat{a}_{11} &= \left( [0.69, 0.76], [0.13, 0.18] \right), \quad \hat{a}_{12} = \left( [0.38, 0.48], [0.25, 0.43] \right) \\
\hat{a}_{13} &= \left( [0.23, 0.44], [0.30, 0.56] \right), \quad \hat{a}_{14} = \left( [0.06, 0.24], [0.56, 0.67] \right) \\
\hat{a}_{21} &= \left( [0.82, 0.87], [0.09, 0.13] \right), \quad \hat{a}_{22} = \left( [0.57, 0.66], [0.08, 0.25] \right) \\
\hat{a}_{23} &= \left( [0.33, 0.65], [0.12, 0.23] \right), \quad \hat{a}_{24} = \left( [0.44, 0.57], [0.30, 0.43] \right) \\
\hat{a}_{31} &= \left( [0.62, 0.82], [0.13, 0.18] \right), \quad \hat{a}_{32} = \left( [0.48, 0.57], [0.16, 0.34] \right) \\
\hat{a}_{33} &= \left( [0.44, 0.54], [0.12, 0.35] \right), \quad \hat{a}_{34} = \left( [0.33, 0.44], [0.12, 0.23] \right) \\
\hat{a}_{41} &= \left( [0.87, 0.91], [0.04, 0.09] \right), \quad \hat{a}_{42} = \left( [0.38, 0.75], [0.08, 0.16] \right) \\
\hat{a}_{43} &= \left( [0.44, 0.57], [0.16, 0.43] \right), \quad \hat{a}_{44} = \left( [0.24, 0.33], [0.12, 0.23] \right) \\
\hat{a}_{51} &= \left( [0.62, 0.69], [0.09, 0.13] \right), \quad \hat{a}_{52} = \left( [0.38, 0.48], [0.43, 0.52] \right) \\
\hat{a}_{53} &= \left( [0.15, 0.33], [0.16, 0.43] \right), \quad \hat{a}_{54} = \left( [0.14, 0.44], [0.46, 0.56] \right)
\end{align*}
\]


Step 2. Utilize the interval-valued intuitionistic fuzzy hybrid geometric (IVIFHG) operator which has associated weight vector \( w = (0.2, 0.30, 0.30, 0.20)^T \), we obtain the overall preference values \( \bar{r}_i \) of the computer network systems \( A_i \) \( (i = 1, 2, 3, 4, 5) \).

\[
\bar{r}_1 = \left( [0.26, 0.45], [0.32, 0.49] \right), \quad \bar{r}_2 = \left( [0.50, 0.67], [0.14, 0.27] \right) \\
\bar{r}_3 = \left( [0.45, 0.57], [0.21, 0.37] \right), \quad \bar{r}_4 = \left( [0.43, 0.61], [0.10, 0.26] \right) \\
\bar{r}_5 = \left( [0.26, 0.45], [0.30, 0.44] \right)
\]
Step 3. Calculate the scores $S\left(\tilde{r}_i\right)\ (i = 1, 2, 3, 4, 5)$ of the overall interval-valued intuitionistic fuzzy preference values $\tilde{r}_i\ (i = 1, 2, 3, 4, 5)$

$$S\left(\tilde{r}_1\right) = -0.05, S\left(\tilde{r}_2\right) = 0.38, S\left(\tilde{r}_3\right) = 0.23$$

$$S\left(\tilde{r}_4\right) = 0.34, S\left(\tilde{r}_5\right) = -0.02$$

Step 4. Rank all the computer network systems $A_i\ (i = 1, 2, 3, 4, 5)$ in accordance with the scores $S\left(\tilde{r}_i\right)\ (i = 1, 2, 3, 4, 5)$ of the overall interval-valued intuitionistic fuzzy values $\tilde{r}_i\ (i = 1, 2, 3, 4, 5)$: $A_2 \succ A_4 \succ A_3 \succ A_5 \succ A_1$, and thus the most desirable computer network system is $A_2$.

5. Conclusion

Wireless Sensor Networks has a wide application, but the security is still an important issue. We extend our work on several hot security-related topics in Wireless Sensor Networks. The first part of our work focuses on building up a trust system in Wireless Sensor Networks to enhance its security. Our trust system is different from traditional trust systems. We suggested a new term "certainty" used in trust system to build trust rating because there is unreliability and uncertainty in Wireless Sensor Networks. The problem of evaluating security of Wireless Sensor Network (WSN) with interval-valued intuitionistic fuzzy information is the multiple attribute decision making (MADM). In this paper, we investigate the multiple attribute decision making (MADM) problems for evaluating the security of wireless sensor networks with interval-valued intuitionistic fuzzy information. We utilize the interval-valued intuitionistic fuzzy hybrid geometric (IVIFHG) operator to aggregate the interval-valued intuitionistic fuzzy numbers corresponding to each alternative and derive the overall value of the alternatives, then rank the alternatives and select the most desirable one(s) according to the score function and accuracy function. Finally an illustrative example has been given to show the developed approach.

6. References


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