A Visual Glove Based on Conformal Geometric Algebra

Peng Chen, Liang Zhou, Yingxue Yao, Shengdong Gao

Abstract

In this paper, a visual glove based on Conformal Geometric Algebra is proposed. To estimate the motion of hand in an intuitive way, Conformal Geometric Algebra is introduced in that it can describe relationship of objects in 3D Euclidean space geometrically. For measuring human hand joints angles with glove like device, the physiological structure of human hand is analyzed and an inverse kinematic model is constructed for obtaining the joint angles of fingers. In the Conformal Geometric Algebra framework, a visual glove without sensors and signal process parts on itself is designed. The features on glove surface are extracted in a binocular computer vision system, and the self-calibration algorithm for the glove is implemented. With the calibrated glove, all the geometric objects for inverse kinematic computing by Conformal Geometric Algebra method are reconstructed in 3D space. With the support of this visual glove, the motion of hand model is demonstrated in the virtual environment.

Keywords: Glove, Computer Vision, Conformal Geometric Algebra, Inverse Kinematic

1. Introduction

Virtual reality is system is widely used in digital entertainment system, medical training system and robot remote manipulation system and manufacturing system. For virtual reality system, human machine interaction is a central function. Participation of hand is needed in most human machine interaction systems because of the flexibility of human hand motion[1]. Glove input device, such as Data Glove and Cyber Glove are widely applied in virtual reality system. Distinct from these whole-hand input devices depending on reporting continues joints angles, a type of choice device called Pinch Glove is also introduced in virtual environment interaction[2]. However, these glove input devices all have sensors or signal process parts attached on the gloves. The architecture of these traditional glove input device is not convenient for their calibration procedure, additionally, the cost and maintenance impede the work-life application of the device. Therefore, a new type of glove device with simple structure, good extensibility and low cost is required for common users of virtual reality systems.

In various realms, a series of new human machine interaction methods implemented by hands are designed. Computer vision is a direct sensing method providing non-contact and non-intrusive natures for motion capture. Ma Deyi et al segment the hand from complex backgrounds[3], and Liu Changzheng et al capture the global hands motion based on depth and silhouette[4]. For detecting finger motions of hand, Ibrahim Furkan Ince at el propose a real Time hand motion detection system based on analysis of finger blobs[5]. A framework of 3D hand tracking and dynamic gesture recognition using a single camera is presented by Ayman El-Sawah and Chris Joslin, they hypothesize the 3D hand posture by geometric and kinematics inverse transformations and apply a model-based method for tracking and recognition[6]. S. Malassiotis and M.G. Strintzis develop a hand posture system relying on 3D sensor that generates a dense range of the scene. The dense vision information is refined to 3D hand geometry directly and the recognition is achieved[7]. F. Malric and A. E. Saddik propose an artificial neural networks method for real-time hand posture recognition. A color-coded glove works in a stereo vision system to obtain meaningful features for computer visual recognition[8]. Jorge Usabiaga and Ali Erol develop an accurate and robust algorithm for the immersive virtual training environment that is called “Virtual Glovebox”. It tracks the 3D position and orientation of an elliptical marker placed on the dorsal part of the hand[9]. Their experimental results illustrate the accuracy and robustness of the marker-based methods. In the research above, it is also revealed that kinematics is crucial in hand gesture recognition and motion estimation. For the articulation of human hand, hand motion is complicated because it not only includes the global motion but also the
translation and rotation of finger joints. Generally, these parameters are assigned to some matrix or quaternion, however, it is very complicated to handle the kinematics involving points, lines and planes. To utilize these geometric objects directly, a new mathematical tool is required. Conformal Geometric Algebra can bring convenience for treatment of this problem[10].

To provide an economical glove input device easily calibrated and estimate the motion of hand in a geometric intuitive way with the glove, a visual glove based on Conformal Geometric Algebra is proposed in this paper. The remainder of this paper is organized as following. In section 2, the introduction of both Geometric Algebra and Conformal Geometric Algebra is given. Then, the physiological structure and kinematic analysis is presented in section 3. In section 4, the self-calibrated visual glove is designed and computing in of joints angle is computed in Conformal Geometric Algebra framework. Finally, the study is concluded in section 5.

2. Geometric Algebra and Conformal Geometric Algebra

Geometric Algebra dates from the 19th century. During this period, mathematicians dedicated to find a perfect presentation of rotation in 3D space. Hamilton found the quaternion generalizing the phase angle to 3D space. At the meanwhile, Grassmann introduce the concept of outer product that is the bivector could be expressed as $a \wedge b$. In 1878, Clifford introduced the concept of "geometric product". It has two parts as (1).

$$ab = a \cdot b + a \wedge b$$ (1)

As is shown above, $a \cdot b$ is the symmetric part of geometric product while $a \wedge b$ is the antisymmetric part of it. The combination of inner product and outer product unified the systems of Hamilton and Grassmann and formed a new algebra system. In this system, geometric objects can be interpreted as algebra entities. Therefore, it could be named Geometric Algebra. Due to the contribution of Clifford, someone also call it "Clifford Algebra" by emphasizing its algebraic attributes.

In Geometric Algebra, $G_{p,q,r}$ is a linear space of $2^n$ dimension with $n = p + q + r$. The blades are the basis of this linear space. For instance, scalars are the grade zero entities, and the vectors are the first grade entities. The bivectors are two grade entities while the k-vectors are k-grade algebraic entities in comparison. If $0, 0, 0 \neq p, q, r$, the space is Euclidean space; if $0, 0, 0 \neq p, q, r$, it is pseudo-Euclidean space; if $r \neq 0$ for only, it is degenerate. For Euclidean space, it has the basis $e_1, e_2, e_3, e_{12}, e_{23}, e_{13}, e_{123}$. The highest grade in Euclidean space is called pseudo scalar $I = e_1 \wedge e_2 \wedge \cdots \wedge e_n$. The linear combination of such k-vectors is defined as multivector. The operation result in the same grades is still this grade while the operation in the different grades is multivector. The multivectors can represent different type of geometric objects such as points, lines, planes, circles, spheres, etc. Therefore, the description of geometric objects in Geometric Algebra is intuitive[11].

As is known, Minkowski plane $G_{1,1}$ has its vector space $\mathbb{R}^{1,1}$ with the orthonormal basis $e_+$ and $e_-$. They has the properties as following.

$$e_+^2 = 1, e_-^2 = -1, e_+ e_- = 0$$ (2)

With the properties in (2), the null vector is defined in (3).

$$e_0 = \frac{1}{2}(e_+ - e_-), e = e_- + e_+$$ (3)

Regard vector $e_0$ as the origin of the coordinate system and e as the point at infinity, it can embed the Euclidean space in a higher dimensional space with the two extra basis vectors. Minkowski plane generates null vectors, and extend Euclidean vector space $\mathbb{R}^n$ to $\mathbb{R}^{n+1,1}$[12]. Thus, the Conformal Geometric $G_{n+1,1,0}$ is generated. For the point x in 3D Euclidean space, the mapping in conformal space is as (4).
\[ X = x + \frac{1}{2} x^2 e + e_0 \]  

(4)

For other geometric entities, the sphere with center in \( X \) and radius \( r \) is as (5).

\[ S = X - \frac{1}{2} r^2 e \]  

(5)

The dual form of the sphere is as (6).

\[ S^*_A = A \wedge B \wedge C \wedge D \]  

(6)

As is shown, the dual form \( S^*_A \) can be calculated directly from points on the sphere. Similarly, the dual form plane is as (7).

\[ P^*_A = A \wedge B \wedge C \wedge e \]  

(7)

Similarly, the dual form of circle is defined as (8).

\[ C^*_A = A \wedge B \wedge C \]  

(8)

With two points and the point at infinity, the line is defined as (9).

\[ L^*_A = A \wedge B \wedge e \]  

(9)

The definition of point pairs only need two points as (10).

\[ PP^*_A = A \wedge B \]  

(10)

All of these geometric objects will be used in the construction of kinematic model in section 3.

3. Kinematic Model for Virtual hand in Conformal Geometric Algebra Framework

Human hand is a complicated articulated system. As is shown in Figure 1, the motion of fingers (thumb is not included) relative to the palm depends on the metacarpophalangeal (MP), the proximal interphalangeal (PIP) and the distal interphalangeal (DIP). With this structure, every finger has four degrees of freedom. For the thumb, there are five degrees of freedom that are two at M, two at MP and one at DIP. The wrist, as the root of the hand, has six degrees of freedom. Totally, there are twenty-seven degrees of freedom for hand motion.

For the motion of fingers, the physiological constrains should be taken into account. The motion of finger joints is not completely free because of the connection of skeleton, muscle and tendon. There is a maximum movement range for each joint of fingers[13], it is shown as Table 1.

![Figure 1. The Physiological Structure of Human Hand](image-url)
Table 1. Maximum movement range of joints

<table>
<thead>
<tr>
<th>Joint of Fingers</th>
<th>Maximum Range (Degree)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DIP of Thumb</td>
<td>75—90</td>
</tr>
<tr>
<td>MP of Thumb</td>
<td>75—80</td>
</tr>
<tr>
<td>DIP of Index</td>
<td>80—90</td>
</tr>
<tr>
<td>PIP of Index</td>
<td>100—110</td>
</tr>
<tr>
<td>MP of Index</td>
<td>90—95</td>
</tr>
<tr>
<td>DIP of Middle</td>
<td>80—90</td>
</tr>
<tr>
<td>PIP of Middle</td>
<td>100—110</td>
</tr>
<tr>
<td>MP of Middle</td>
<td>90—95</td>
</tr>
<tr>
<td>DIP of Ring</td>
<td>80—90</td>
</tr>
<tr>
<td>PIP of Ring</td>
<td>110—120</td>
</tr>
<tr>
<td>MP of Ring</td>
<td>90—95</td>
</tr>
<tr>
<td>DIP of Little</td>
<td>80—90</td>
</tr>
<tr>
<td>PIP of Little</td>
<td>100—110</td>
</tr>
<tr>
<td>MP of Little</td>
<td>90—95</td>
</tr>
</tbody>
</table>

The kinematic chain of finger includes MP, PIP and DIP in Figure 1. If the length of these three segments are known, the joint angles could be computed by inverse kinematics method when the position of finger tip and MP are captured. To construct the Conformal Geometric Algebra kinematic model, the geometric algebra software tool CLUCalc is utilized for the simulation of kinematic chain. The geometric objects in conformal space can be generated in CLUCalc. In the software, the finger is modeled as Figure 2. Let the length of MP, PIP and DIP are \( l_1, l_2 \) and \( l_3 \) respectively, and the tip of the three segments are point \( X_1 \), point \( X_2 \) and point \( X_3 \). Set the MP of the finger on the origin, the segment of MP is vertical to the plane \( xz \). Suppose all the four points \( X_1, X_2, X_3 \) and \( X_n \) are in the same plane \( P_f \). Thus, the plane in conformal place is as (11).

\[
P_f = X_1 \wedge X_2 \wedge X_3
\]  

(11)

The line segment represented MP could be computed by the origin and \( X_1 \) and let it be \( L_1 \), while the \( L_2 \) and \( L_3 \) are unknown element. If the point \( X_n \) is obtained, \( L_1 \) and \( L_3 \) are fixed correspondingly. There are two geometric constrains for \( X_n \). The first one is \( X_n \) on both the sphere \( S_1 \) and \( S_2 \) as following.

\[
S_1 = P_2 - \frac{1}{2} r_2^2 e
\]  

(12)

\[
S_2 = P_3 - \frac{1}{2} r_3^2 e
\]  

(13)
Factually, the two spheres are intersected as a circle as the Figure 3 shown.

![Figure 3. Circle intersected by two spheres](image)

The circle with yellow color in Figure 3 is as (14).

$$C = S_1 \wedge S_2$$  \hspace{1cm} (14)

The second constrain is that $X_n$ is on the plane $P_f$ as supposed. Thus, $X_n$ is one of the points pair generated by the intersection of $C$ and $P_f$ as (15).

$$PP_n = (C \wedge P_f)^*$$  \hspace{1cm} (15)

For the points pair, $X_n$ would be the one satisfying the angle range of finger joints physiologically. Therefore, $L_2$ and $L_3$ are construct for all the end points of them are deduced. The angle between them could be computed as (16) according to the formula in [14].

$$\text{Angle}_{23} = \arccos \frac{L_2^* \cdot L_3^*}{\|L_2^*\| \|L_3^*\|}$$  \hspace{1cm} (16)

### 4. Design of Visual Glove and Implementation in Virtual Environment

The visual glove proposed here is a type of glove working in a computer vision system. For the requirement of 3D information, the computer vision system here is a binocular one. The binocular system is shown as Figure 4. Simultaneous, the basic elements in Conformal Geometric Algebra are points, lines, planes, circles and spheres. For these reasons, the surface of the glove should have the features closed to these elements and easy to extract by binocular vision system. Consequently, the glove is designed as a five-colored one according to every finger and with the features of ellipse, line and hexagon meshes for the capture of computer vision system. In the binocular system, the left and right images of the glove is shown as Figure 5.
The designed glove is adept to different users for its elasticity. After the computer vision system starts, the glove can calibrate itself by measuring the dimension of user hand during working. The main step of the computer vision algorithm is as follows.

Step 1: Input the left and right images.
Step 2: Convert the images into HSV format.
Step 3: Separate the different fingers by color in HSV space.
Step 4: Extract the features of corners.
Step 5: Search the ellipse features on tips, lines features on finger segments and hexagon meshes features from the corners obtained.
Step 6: Construct correspondence of features between two images.
Step 7: Compute the 3D position of every segment of finger, and obtain the lines of finger segments by the intersection of planes through optical centers and line segments on the two images.
Step 8: Obtain the initial position of every joints and compute the length of every finger segments.
Step 9: Compare the length of segments between the frames until the error is accepted.
Step 10: Self-calibration of the glove is finished and the length of every finger segment is obtained.

With the calibrated glove, the motion of fingers could be estimated by the inverse kinematic method based on Conformal Geometric Algebra in section 3. The result of motion estimation is mapped to the hand model in virtual environment as Figure 6 shown.
5. Conclusion

A visual glove based on Conformal Geometric Algebra has been designed and implemented in this paper. Conformal Geometric Algebra has been introduced to estimate the motion of hand in a geometrically intuitive way. Physiological structure of human hand has been analyzed and an inverse kinematic model has been constructed for obtaining the joint angles of fingers. In the Conformal Geometric Algebra framework, the visual glove without sensors and signal process parts on itself works in a binocular computer vision system, and the self-calibration algorithm for the glove is presented. With the calibrated glove, all the geometric objects for inverse kinematic computing by Conformal Geometric Algebra method are reconstructed in 3D space. With the support of this visual glove, the motion of hand model is demonstrated in the virtual environment. In future work, the robustness and the real-time performance of computer vision algorithm will be improved, and the virtual hand model will be applied in object manipulation for human machine in specified fields.

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7. References


