A Novel Image Edge Detection Algorithm based on Prewitt Operator and Wavelet Transform

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Abstract

In this paper, the improved version of Prewitt operator edge detection algorithm proposed which is based on the soft-threshold wavelet de-noising is aimed at processing the image with Gaussian white noise. A number of experiments show that this algorithm adds some advantages of wavelet de-noising, such as that the differential nature of the wavelet function can highlight the discontinuous edge of the image, the multi-resolution of wavelet transform overcome the contradiction between reduction of noise and the precision of edge positioning in the field of neighborhood selection. At the same time the wavelet transformation has the function of band-pass filter, and it can extraction the different levels of information. The improvement of algorithm combined by soft-thresholding wavelet de-noise and the conventional prewitt operator is a much better method than the conventional edge detection operator when it detects the edge of the image with White Gauss Noise.

Keywords: Edge Detection; Hard-Threshold De-Noising; Soft Threshold De-Noising; Wavelet Transform; Median Filter; Prewitt Operator

1. Introduction

The edge detection of the classical operator is first analysis in this paper. It is found that Prewitt operator and Canny operator have a satisfactory results in edge detection on noise-free images, but the edge detection results of all the classical operators are all unsatisfactory after the Gaussian white noise is added into images. In order to overcome the contradictions between noise suppression and edge select positioning accuracy in the field of traditional algorithms. An improved edge detection algorithm, which is based on the combination between a soft-threshold wavelet de-noising and Prewitt operator, is presented in this paper. By a large number of experiments, we can find that this operator is obviously better than the operators like the Prewitt operator based on median filtering, the Prewitt operator based on hard-threshold wavelet de-noising and the ordinary Prewitt operator based on Wavelet Denoising. The algorithm proposed in this paper add the multi-resolution characteristics of Wavelet and the band-pass filter characteristics of Wavelet transform into the the Prewitt operator perfectly. And the soft-threshold is continuous in \( y = \pm \hat{x} \), so that the details of the edge of the image is kept well, and make the image edge detection more exactly, and the effect that Gaussian white noise makes on the edge detection results can be restrained in the maximum extent possible.

Edge includes the most important information in the image, such as the edges which are used for labeling the abrupt points and the points with a significant variation in gray level, and can provide the information of the object’s position, so the edge detection plays an important role in applying of image analysis. Edge detectors of some kinds, particularly step edge detectors, have been an essential part of any computer vision systems. The edge detection process serves to simplify the analysis of images by drastically reducing the amount of data to be processed, while at the same time preserving useful structural information about object boundaries. For decades, a lot of approaches have been developed for edge detection. The usual methods are: Sobel operator [1], Prewitt operator [2], Marr operator [3], Canny operator [4], and so on. These methods need two steps: edge gradient map calculation and edge detecting by a proper gradient magnitude threshold method, and make use of second-order derivatives operator or appropriate threshold value obtain image edge. Huang et al.[5] proposed EMD firstly for the processing of non-stationary functions. It is more like an algorithm (an empirical approach) that can be applied to a data set, rather than a theoretical tool. The key part of this method is the “empirical mode decomposition” with which any complicated data set can be decomposed into a finite and often a small number of intrinsic mode functions (IMF). This decomposition method is adaptive, and, therefore, highly efficient. Since the decomposition is based on the local characteristic time scale of the
2. PREWITT OPERATOR

Prewitt operator reaches extreme detection edge in the edge and makes noise smooth by grayscale difference of adjacent pixels from top to bottom, left and right. Because there are many obvious difference between the gray value of edge points' pixel and the gray value of its neighborhood points' pixel, so that in practice we usually detect the edge of the image by differential operator and template matching method. The so-called template matching method is to extract the edges of objects by understanding the matching score between objects and template. In other words, we should first choose some representative standard model, and then compare some images with these templates, last choose the most alike.

While the method that Prewitt operator combine templates to research on edge detection cost a large computation and processing time for pictures. So it is not fit the Real-time Image Processing System. We must improve the operator so that it can make a better result in processing speed and save time for the subsequent image-processing.

Prewitt operator have eight templates corresponded with boundary. Every template makes the maximum response to some special edge orientation. The maximum of all the eight direction are used to be the export of edge amplitude image.

The eight templates of Prewitt operator are respectively shown in Figure 2.

\[
\begin{array}{ccc}
1 & 1 & 1 \\
1 & -2 & 1 \\
-1 & -1 & -1 \\
\end{array}
\quad
\begin{array}{ccc}
1 & 1 & 1 \\
1 & -2 & 1 \\
-1 & -1 & -1 \\
\end{array}
\quad
\begin{array}{ccc}
1 & 1 & -1 \\
1 & -2 & 1 \\
-1 & 1 & -1 \\
\end{array}
\]

(a) (b) (c)

\[
\begin{array}{ccc}
1 & -1 & -1 \\
1 & -2 & 1 \\
-1 & 1 & -1 \\
\end{array}
\quad
\begin{array}{ccc}
-1 & -1 & -1 \\
-1 & -2 & 1 \\
1 & 1 & 1 \\
\end{array}
\quad
\begin{array}{ccc}
-1 & 1 & -1 \\
-1 & -2 & 1 \\
1 & 1 & 1 \\
\end{array}
\]

(d) (e) (f)

\[
\begin{array}{ccc}
-1 & 1 & 1 \\
-1 & -2 & 1 \\
-1 & 1 & 1 \\
\end{array}
\quad
\begin{array}{ccc}
1 & 1 & 1 \\
-1 & -2 & 1 \\
-1 & 1 & -1 \\
\end{array}
\quad
\begin{array}{ccc}
1 & 1 & 1 \\
-1 & -2 & 1 \\
-1 & 1 & -1 \\
\end{array}
\]

(f) (g) (h)

Figure 1 eight template of Prewitt operator

Suppose in the image, the grayscale of a point Q and its 3 x 3 -area is shown in the follow image 2. We suppose respectively that \( a_i \) (\( i = 1,2,3,4,5,6,7,8 \)) is the grayscale of point Q after the image are processed by the first i kinds of template processing of Prewitt operator.

Such as using the template (c), then
After be processed the gray value \( W \) of point Q is:

\[
W = \max(a_i), i = 1, 2, 3, 4, 5, 6, 7, 8
\]

We count the pixel grayscale by the above Prewitt eighttemplate method, and analyze the calculation adopting Prewitt operator to process image. Use Prewitt operator 8 templates to process an \( N \times N \) image. The calculation required in the method above is that the number of addition is \( 64N^2 \) times and the number of multiplication is \( 8N^2 \) times. The calculation is so large that it affects the image processing speed. This is very inconvenient to the image systems which require real-time and speed.

3. Based on Soft-Threshold Wavelet De-Noising Combining with The Prewitt Operator Edge Detection Algorithm

Prewitt operator carries out edge detection to an image in 8 directions, and let the maximum direction response be the edge of the edge magnitude image, it also has the smoothing effect on the noises. For the traditional edge detection operator, there is a contradiction between the ability to smooth noises and the ability to locate the edge. In order to overcome this deficiency, this paper combines the method of soft-threshold wavelet de-noising and Prewitt operator to propose a new and improved edge detection operator. This method is for the images containing Gaussian white noises. It can not only filter out the noise signals very well, but also extract image edge correctly to achieve a satisfactory effect on edge detection and pave the way for the follow-up technologies.

3.1 The method of threshold-based wavelet de-noising

In order to avoid the shortcomings above, we can set the threshold. When the high-frequency coefficient is less than or equals the threshold, we set it to zero; on the contrary, the ones greater than the threshold will be retained. Following that method, we can achieve the purpose of keeping the details and removing the noise.

Threshold filtering is used to filter Gaussian white noise in the signal. Due to the energy of Gaussian white noise in the wavelet transform domain focused on a smaller amplitude of the wavelet coefficients, we can set the threshold value according to this feature and in a general way we set the wavelet coefficient to zero when it is below the threshold value so that the noise in signal can be effectively inhibited. Wavelet transform has a capability of "focusing". It can focus the energy in different frequency domain of the signal on a small number of coefficients which get from wavelet transform. Relatively speaking, the value of these coefficients are necessarily greater than those ones whose energy in the wavelet transform domain spread over a large number of wavelet coefficients of either signals or noise. It means that we can select each threshold in the appropriate scale so as to remove low amplitude noise and the undesired signals, then wavelet transform to get reconstructed signal.

Wavelet Bases are not unique in the wavelet transform. It depends on the application and the different de-noising method. In order to obtain restored images after de-noising, we often adopt linear de-noising method. It is that after wavelet transform, the wavelet coefficients of image with noise...
directly get rid of some of the high frequency components, but it has already been mentioned that it is easy to lose the edge information of image following this method. So it is always not clear at the edge of the image, which would affect the precision of edge detection.

Therefore, this article selected two kinds of wavelet thresholding de-noising methods based on non-linear. Firstly we fetch one image which contains Gaussian white noise and handle with it by wavelet transform to the four components, respectively indicated by  \( W^1_{j,k,l} \),  \( W^2_{j,k,l} \),  \( W^3_{j,k,l} \) which represent second high-frequency and by  \( H_L^{j,k} \),  \( H_H^{j,k} \),  \( H_H^{j,k} \) which represent high frequency of the wavelet coefficients, where \((K, L)\) expressed as a two-dimensional wavelet coefficients,  \( j = 1,2, \ldots \) for the decomposition level. We get the last low-frequency component  \( W^0_{j,k,l} \) that is:  \( L_L^{j,m} = W^0_{j,k,l} \). We presume \( \lambda \) is the threshold calculated, and there are two kinds of image denoising ways based on threshold as followed:

The first way: Non-linear shrinkage threshold

Shrink the threshold of the wavelet transform coefficients:

\[
W^i_{j,k,l} = \begin{cases} 
W^i_{j,k,l} - \frac{\lambda}{2^i}, W^i_{j,k,l} > \lambda \\
0, & W^i_{j,k,l} \leq \lambda \\
W^i_{j,k,l} + \frac{\lambda}{2^i}, W^i_{j,k,l} < -\lambda 
\end{cases}
\]  \( \lambda \)  \( (2) \)

Where,  \( j = 1, 2 \ldots \) for the decomposition level,  \( i = 0,1,2,3 \).

The second way: Direct threshold

In the wavelet coefficients domain, we directly judge the wavelet transform coefficients  \( W^i_{j,k,l} \) (i = 0, 1, 2, 3;  \( j = 1, 2 \)) by threshold operation, so there are two ways:

1. Hard-threshold method:

   1) Firstly, handle with the images containing Gaussian white noise by wavelet transform.

   2) Secondly, except the coarsest scale, the signal will consider all the details by threshold operation. When the position value is greater than the wavelet threshold, we retain the original value, otherwise set it to zero.

\[
W^i_{j,k,l} = \begin{cases} 
W^i_{j,k,l} - \frac{\lambda}{2^i}, |W^i_{j,k,l}| \geq \lambda \\
0, & |W^i_{j,k,l}| \leq \lambda 
\end{cases}
\]  \( \lambda \)  \( (3) \)

3) Thirdly, reconstructing after wavelet transform, we find the filtered image.

2. Soft-threshold method

1) Firstly, we get the wavelet transform coefficients handling with the images containing white noise by wavelet transform.

2) Secondly, we handle with the details of the signal by threshold operation. When the position value is greater than the wavelet threshold, do further processing as followed, where  \( \text{sgn}(x) \)  is on behalf of sign function.

\[
W^i_{j,k,l} = \begin{cases} 
\text{sgn}(W^i_{j,k,l}) \times |W^i_{j,k,l}| - \lambda, |W^i_{j,k,l}| \geq \lambda \\
0, & |W^i_{j,k,l}| \leq \lambda
\end{cases}
\]  \( \lambda \)  \( (4) \)
Otherwise we set it to zero.

3) Wavelet coefficients $W_{ij}^{g}$ are given valuation $\hat{W}_{ij}^{g}$, and after processing, we operate wavelet inverse transform in the wavelet coefficients to reconstruct the image signal, find the filtered values. (see the following figure)

(a) hard-threshold de-noising filter 
(b) soft-threshold de-noising filter

Figure 3. The comparison between hard-threshold and soft-threshold de-noising filter

3.2 The improved Prewitt operator

The Prewitt operator is a method which uses airspace differential convolution or an operation similar to convolution. The substance is using a partial differential operator to do convolution to the image, and then using the amplitude extremism of the first derivative or the amplitude zero-crossing point of the second derivative to detect the edge. This classic operator has been used widely in the field of edge detection. However, it has many shortcomings itself. For example, after the image signal being polluted by a large number of Gaussian white noises, Prewitt operator uses the significant difference of the gray value between the pixel of the edge point and the pixels of its nearby points, and then uses the differential operator and the method of template matching to detect the image edge. The pixel gray value between the points of the Gaussian noises and the useful signals in the image is very different. Therefore, the range value of the first derivative for the edge of adjacent pixels significantly changes; Prewitt operator can easily detect the noise signals and have a certain degree of smoothing effect on the image. The detected result will amplify the noise signals and blur the useful signals of the edge. This will affect the image detection results significantly. This paper proposes an improved Prewitt operator for the images which contains white Gaussian noises. This method will combine the advantages in removing Gaussian white noises based on soft-threshold wavelet de-noising and the classical edge detection technology based on Prewitt operator. It can not only remove the noise signals, but also detect the image edge very well.

From 3.1 and 3.2, we know that the de-noising effect which uses the wavelet de-noising based on setting high frequency coefficients to zero is obvious, and the calculation is relatively small. However, the signals reconstructed after using the method of setting high-frequency coefficients to zero Mandatory will miss details and the image will become blurred. However, Prewitt operator can not distinguish the edge signals of the image and the test result is not good. Hard-threshold de-noising can be better to retain the edge details of the image, and the result is much better than that obtained by the method of setting High-frequency coefficients to zero. But the result obtained by hard threshold is cruder than that obtained by soft threshold. The main result is that, Hard-threshold de-noising filter in Figure 3 is discontinuous at the place where $y = \pm A$ while Soft-threshold is continuous. As an extension of hard-threshold de-noising, soft-threshold de-noising can obtain a quite satisfactory result. So this paper selects the algorithm which combines soft-threshold de-noising and Prewitt operator to do edge detection to the image.

The algorithm:

1) Do wavelet decomposition to the image matrix and obtain the wavelet coefficients with noises.
2) Process the HL, LH and HH wavelet coefficients obtained by decomposition according to the formula (4) and the Low-frequency coefficients are the same.
(3) Do wavelet inverse transform to the wavelet coefficients and reconstruct the image matrix obtained after de-noising.

(4) Define the template edge coefficients according to the Prewitt operator template in Figure 1.

(5) After given the Prewitt edge detection operator template, every image pixel will do a convolution with the template, and obtain the gradient of this point. The amplitude of the gradient is the output of this point. At the end we get the image of the edge detection.

4. Experimental Results and Analysis

In this section we take the image "lena" with a Gaussian white noise as the original image. The first use of classical edge detection operator (including the Sobel operator, Prewitt operator, Laplacian operator, as well as the canny operator) on the noise image edge detection, and compare the test results. Second, using median filtering method, based on the general threshold wavelet denoising methods and algorithms based on morphology, and the combination of classical operators on the noise image edge detection. Compared with the results of experiments above, it exist deficiencies in the variety of algorithms themselves. However, in the paper we bring up based on the softthreshold wavelet de-noising combining with the Prewitt operator. It can get optimal results when detect the Lena image edge with a Gaussian white noise. We take the image "lena" with a Gaussian white noise as the original image. First use the classical edge detection operator for image edge detection. As shown in Figure 4, 5, 6, 7 and 8.

**Figure 4** Lena image with Gaussian white noise

**Figure 5** Sobel edge detection operator

**Figure 6** Prewitt edge detection operator
We can see from the above picture, the image with Gaussian white noise. When the classical edge detection operator against the image with Gaussian white noise on the edge detection, the noise points of the image have also been detected, meanwhile the image edge is not clear. In particular the canny operator, Sobel operator and the laplacian operator in a certain degree change the contrast of the image, and edge detection is also not clear enough. Compared to other operators for the Prewitt operators detect the image with the Gaussian white noise, the effect of image edge detection more obvious.

However, due to the traditional edge detection operator noise smoothing ability and the ability to locate the edge of the existence of contradictory, In order to overcome this deficiency, The best way is to de-noising effect of a number of obvious ways to mix with those operators, can effectively filter out the noise can also try to keep the edge of the image details, while the role of traditional edge detection operator characteristics.

This paper selects several common de-noising methods which combined with the traditional detection operator. Compared with results of the experiments, as shown in Figure 9, 10, 11 and 12.
Figure 9 is based on the method which combines the median filtering and the Prewitt operator. Median filtering can not only eliminate the peak pulse and high-frequency noise, but also inhibit step and ramp without causing phase mutations. Therefore compared with figure 9, figure 6 tell us that the Gaussian white noise in the picture is effectively inhibited, but because the median filtering will produce a certain smoothing effect for the image with noise, the detected edge of the image is fairly vague. It is also the biggest shortcoming of this method.

Figure 9 is based on the method which combines the ordinary wavelet de-noising and the Prewitt operator. The advantage of this method is that it can filter the white Gaussian noises in the high-frequency parts of the signal to the greatest extent. Compared to the other methods, we can see that the noise signals can be filtered cleaner. However, when we do the first-order derivative to the edge of the image, its peak hopping reduced or eliminated, it is just because of setting high-frequency coefficients to zero makes the result of the edge detection very vague and some of the edge details ignored.

Figure 10 is based on the method which combines hardthreshold wavelet de-noising and the Prewitt operator. The result obtained by this method is far better than that obtained by the ordinary wavelet de-noising. And this method can retain the details of the image edge very well. However, some of the details in the image are still a little rough.

Figure 11 is a method based on the combining of hard thresholding wavelet de-noising and Canny operator, it detects noise-free images by using Canny operator, and the detection results are very well, at the same time its algorithm is simple and computing speed is comparatively fast. But figure 10 shows that when it processes images with white Gaussian noise, the detection results is far from satisfactory. Wavelet de-noising is added into figure 12, and combines with canny operator, and still can not get satisfactory results. Because the sensitivity of canny operator detecting edge detail is higher than other traditional detection operator, so that canny operator has a high sensitivity when it detects noises, and the edge detail of the image will be howled down by many white noise signals.

Through the comparison of experimental images, the detected results followed the ways above are not ideal because of their own shortcomings. In order to achieve the purpose of filtering noise better, retaining the details of the image edges and enhancing the results of edge detection, in this paper, I propose a method of improved Prewitt operator edge detection based on the soft threshold wavelet de-noising, this paper presents a based on soft-threshold wavelet denoising combining with the Prewitt operator edge detection algorithm, as shown in the Figure 13.
Figure 13 Soft-threshold wavelet de-noising-Prewitt operator

The methods used on map are based on the soft-threshold wavelet de-noising combining with the Prewitt operator. Contrast to Figure 6, Figure 9, Figure 10 and Figure 11, This method is to filter out noise better than hard-threshold denoising and the Prewitt operator a combination of methods. The reason is that soft-threshold de-noising filter in $y = \pm \lambda$ is that continuous, while the hard-threshold de-noising filter in $y = \pm \lambda$ is that discontinuous. So this method in detecting the edge of the details better. And this method is to retain the edge of the details in the image is better than another based on the median filter combining with the Prewitt operator and based on general Wavelet denoising combining with the Prewitt operator. This method is used to filter out noise, selectivity filter, rather than all at the same time to filter out high frequency noise signals, thus avoiding losing a lot of useful signal. Compared with results of the experiments, We can see that the based on soft threshold wavelet de-noising combining with the Prewitt operator edge detection algorithm compared to other methods, in the de-noising, retain edge detail and accuracy in detection is the best.

5. Conclusion

Moreover, the edge detection result obtained by median filter or the method which combines the ordinary wavelet de-noising and the traditional detection operator isn't better than that obtained by the method proposed in this paper too. So when we detect the images which contain the white Gaussian noises, the result obtained by the improved Prewitt operator proposed in this paper is more satisfactory. However, the process of some noise signals is still not ideal, and this is the emphasis which needed improved in the future.

6. References