An Improved Semi-blind Joint Data Detection and Channel Estimation Algorithm for MIMO-OFDM System

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Abstract

In this paper, a new Semi-blind joint Grover’s Quantum Search (GS) based data detection and Space-Alternating Generalized Expectation-maximization (SAGE) channel estimation algorithm for Multiple-Input Multiple-Output (MIMO)-Orthogonal frequency division multiplexing (OFDM) system is proposed. According to the training symbols inserted in the head of sub-frame, we get the initial estimation of joint algorithm with the Maximum a posteriori (MAP) estimator. Then, an iterative optimization loop which joints the SAGE’s E&M steps and GS based data detection is employed. In the iteration process, we apply channel estimation of the previous transmitted symbols to initial estimation in the current transmitted symbols with GS based data detection, update channel estimate and data detection until converge. All transmitted symbols can be updated in turn. The simulation results show that our proposed joint algorithm for MIMO-OFDM system have low Bit Error Rate (BER), and also have lower complexity than joint Maximum-Likelihood (ML) based data detection and SAGE based channel estimation algorithm.

Keywords: Multiple-Input Multiple-Output, Orthogonal Frequency Division Multiplexing, Grover’s Quantum Search, Space-Alternating Generalized Expectation-Maximization

1. Introduction

For the need of higher bandwidth efficiency and performance over very challenging channels that may be time-selective and frequency-selective in the next generation wireless communication systems, the combination of MIMO and OFDM has attracted much research interest. The systems with MIMO techniques can achieve a significant increase in the bandwidth efficiency and an improvement of the transmission reliability over flat fading channels by diversity gain and coding gain. However, if the channels are non-flat fading channels such as frequency-selective multi-path channels, the function of MIMO system will be destroyed. The systems with OFDM techniques can divide a frequency-selective channel into many narrow parallel frequency flat subchannels. It can increase the symbol duration, and the ISI (Inter Symbol Interference) caused by multipath can be reduced or eliminated. Hence using OFDM techniques in MIMO systems, the channel impulse response can be considered to be flat within each sub-carrier. MIMO systems with OFDM techniques can be effectively used in non-flat fading channels, and it is a suitable choice for future high-data rate wireless systems [1].

Channel estimation algorithm plays an important role in MIMO-OFDM system. The channel state information (CSI) can be extracted based on channel estimation algorithm for MIMO-OFDM system receiver. If the receiver can get an accurate CSI, it can detect the transmitted signal from each transmit antenna with data detection effectively. Hence an efficient channel estimation which can estimate an accurate CSI at the receiver is necessary for MIMO-OFDM system. The channel estimation algorithms for MIMO-OFDM system include blind channel estimation algorithms and Semi-blind channel estimation algorithms. Blind channel estimation algorithms use the received signals and the stochastic information of transmitted and received signals to estimate the channel coefficients. It is always based on subspace [2]. The most widely used is Semi-blind channel estimation algorithms. In Semi-blind channel estimation algorithms, Least Square (LS) channel estimation using pilot symbols or training sequences [3-4] and minimum mean square error (MMSE) channel estimation [5-7] has been proposed for MIMO-OFDM systems. [7] described a general Minimum Mean Square Error (MMSE) channel estimation algorithm for MIMO-OFDM systems which may be pilot-symbol-assisted systems, pilot-embedded systems, or blind systems. The complexity of such algorithms is low, and it not only estimate the channel information using training sequence, but also use the unknown information...
sequence information to further improve the accuracy of channel estimation. It has attracted much attention in the study of Channel Estimation for MIMO-OFDM systems. Our proposed SAGE based channel estimation algorithm is one of Semi-blind channel estimation algorithms.

According to the CSI extracted based on channel estimation, receiver can detect the transmitted signal from each transmit antenna with data detection effectively. MIMO data detection techniques can also be used in MIMO-OFDM system. Conventional MIMO data detection techniques are the ML detector, Minimum Mean Square Error (MMSE) detector, Zero-Forcing (ZF) detector, detector based on Successive Interference Cancellation (SIC) or Parallel Interference Cancellation (PIC), QR-decomposition detector and the Bell Labs layered space–time (BLAST) detector[8-12]. ML detector is the optimal detection scheme, but with a high complexity. Hence how to reduce the complexity of ML detector has caused much attention. The Quantum computing [13] based on quantum circuits is a combination of quantum theory and information science. Quantum evolutionary-inspired algorithm [14-15] and Grover’s quantum search (GS) algorithm [16] are the most commonly used algorithms based on quantum computing. The GS algorithm can increase the probability amplitude of solutions while reduce the probability amplitude of non-solutions by Grover’s iterative process in the problem of searching for optimal solution. Hence Grover’s quantum search algorithm can be used to extract statistics, such as the minimal element, from an unordered data set, more quickly than is possible on a classical computer. Apply it in data detection is necessary for low complexity.

A Zero-Forcing (ZF)-based SAGE algorithm for MIMO system is proposed in [17]. It can track the channel variation and has low computational complexity. It can achieve the better Bit Error Rate (BER) than ML data detection with training symbols. By setting the new observed data as the incomplete data and the complete data, [18] proposed a MAP-based SAGE channel estimation and data detection joint algorithm for MIMO system. But all of them did not analyze the application in MIMO-OFDM system, and they always have a high computational complexity. For the application in MIMO-OFDM system and lower computational complexity, we propose a semi-blind joint Grover’s quantum search based data detection and SAGE based channel estimation algorithm for MIMO-OFDM system according to [17, 18].

The rest of this paper is organized as follows. Section 2 presents the system model. The general Grover’s quantum search algorithm and SAGE algorithm are presented in Section 3. In section 4, we present our GS based data detection algorithm and SAGE based channel estimation algorithm in detail. The application of joint GS based data detection and SAGE based channel estimation algorithm is presented in section 5. In section 6, we evaluate the performance of our joint algorithm. Section 7 draws a conclusion about the whole article.

Notations: The boldface letters will be used for matrices and column vectors; $\mathbb{E}\{\cdot\}$ denotes expectation; $[\cdot]^\dagger$ denotes the transpose operation; $\cdot^\dagger$ denotes the complex conjugate; $\otimes$ denotes the tensor multiplication; $\mathbf{I}_P$ is the identity matrix of size $P \times P$.

2. System Model

As is shown in Fig.1, we consider a MIMO-OFDM system with $N$ transmits antennas and $M$ receives antennas.

![Figure 1. A basic structure of spread MIMO-OFDM system](image)

At the transmitter, the modulated symbols are encoded into $N$ Low-speed parallel data streams, corresponding to the transmit antennas after MIMO encoder. Before IFFT, the corresponding pilot
symbols (or training symbols) are inserted as the preamble followed by MIMO-OFDM symbols. They are used for time/frequency synchronization and channel estimation. The inverse FFT (IFFT) can be used to act as a modulator at transmitter and the FFT to act as a demodulator at receiver in discrete time. Each output of the IFFT is converted to a serial sequence and a cyclic prefix (CP) is added to avoid ISI. The OFDM signal is transmitted over the pass-band RF channel, received, and down-converted to base band. At the receiver, after CP removal and FFT, our Joint algorithm is performed to estimate the CSI (channel state information) by the pilot symbols (or training symbols) and detect the transmitted symbols according to the CSI. CSI is required in MIMO-OFDM for MIMO coding at the transmitter and signal detection at receiver. Its accuracy directly affects the overall performance of MIMO-OFDM systems. The pilot symbols (or training symbols) are also used for time/frequency synchronization which is very important to the FFT.

According to Fig. 1, in addition to the training symbols, \( \mathbf{X}_n = [X'_1, \cdots, X'_N]^T \) \((n = 1, \cdots, N)\) can be expressed one frame of the transmitter at the \( n \)-th transmit antenna. \( X'_n \) is the \( L \)-th subcarrier of OFDM symbol at the \( n \)-th transmit antenna. The \( K \)-th subcarrier of OFDM symbol at the \( m \)-th receive antenna can be expressed as

\[
Y^m = \sum_{n=1}^{N} H_{n,m}^k X_n^k + W^k \quad (k = 1, \cdots, L)
\]

(1)

Where \( H_{n,m}^k \) is the channel gain at the \( K \)-th subcarrier of OFDM symbol from the \( n \)-th transmit antenna to the \( m \)-th receive antenna. \( W^k \) is the additive White Gaussian noise (AWGN) with zero mean and variance \( \sigma^2 \).

3. The general Grover’s Quantum Search algorithm and SAGE algorithm

3.1. The general Grover’s Quantum Search algorithm

3.1.1. A Brief introduction of quantum computing

Grover’s quantum search algorithm is based on quantum computing. Different from traditional computing, the quantum computing uses the orthonormal basis \( |0\rangle \) and \( |1\rangle \) to express the classical bit values 0 and 1 respectively. A quantum bit (qubit) can be expressed in a linear superposition of the basic \( |0\rangle \) and \( |1\rangle \) [13]:

\[
|\phi\rangle = \alpha |0\rangle + \beta |1\rangle
\]

(2)

Where \( \alpha \) and \( \beta \) are complex numbers, and they must satisfy the requirement:

\[
|\alpha|^2 + |\beta|^2 = 1
\]

(3)

A quantum system with \( n \) qubits is always called quantum register (qregister). If a qregister has \( n \)-qubits, the state of this qregister can be expressed as a superposition state:

\[
|\psi\rangle = |\psi_0\rangle \otimes |\psi_1\rangle \otimes \cdots \otimes |\psi_{2^n-1}\rangle = \sum_{i=0}^{2^n-1} \omega_i |\psi_i\rangle
\]

(4)

Where \( |\psi_i\rangle (i = 0, 1, \cdots, 2^n - 1) \) is the Hilbert space basis, \( |\psi\rangle \) is the unit vector of \( 2^n \)-dimensional Hilbert space, \( \omega_i \) is the probability amplitude of \( |\psi_i\rangle \), and \( |\alpha|^2 + (\sum_{i=0}^{2^n-1} |\omega_i|^2) = 1 \) is the probability amplitude of \( |\psi_i\rangle \) when \( |\psi\rangle \) is measured.
3.1.2. Grover’s Quantum Search algorithm

The Grover’s Quantum Search algorithm based on quantum computing have there steps as follows [16]:

1. Initialize qregister \( |0\rangle^\otimes n \) by Walsh-Hadamard transformation \( W = H^\otimes n \). Hence we can get an equal superposition state of \( n \)-bit qregister \( |s\rangle \).

\[
H^\otimes n |0\rangle^\otimes n = \frac{1}{\sqrt{2^n}} \sum_{x=0}^{2^n-1} |x\rangle = |s\rangle \tag{5}
\]

2. Carry out the Grover iteration \( G = UO \) about \( \frac{\pi}{4} \sqrt{2^n} \) times. It can be divided into two steps.

   **Step1:** Apply the “Oracle”.
   
   The aim of Oracle is to mark the target state. The Oracle transforms the search problem in an unsorted quantum database into a decision problem by defining a function \( f(x) \). Hence if want to search a target element from a space with \( 2^n \) elements which is indexed by \( x \) \((x=0, 1, \ldots, 2^n-1)\). The Oracle can calculate the value of the function \( f(x) \) and compare it with conditions.

   \[
f(x) = \begin{cases} f(x) = 1, & \text{if } x \text{ satisfy the conditions} \\ f(x) = 0, & \text{otherwise} \end{cases} \tag{6}
\]

   It can be expressed by a unitary operator \( O \):

   \[
   |x\rangle \xrightarrow{O} (-1)^{f(x)} |x\rangle \tag{7}
   \]

   **Step2:** Apply the \( U \) operation on the equal superposition state \( |s\rangle \).

   \[
   U = WRW = 2 |\varphi\rangle\langle \varphi | - \mathbf{I} \tag{8}
   \]

   Where \( R = 2 |0\rangle\langle 0 | - \mathbf{I} \), \( |\varphi\rangle = \frac{1}{\sqrt{2^n}} \sum_{x=0}^{2^n-1} |x\rangle \).

3. Measurement. After Grover iteration, take a measurement of the qregister state, we can find a solution of search problem \( |x\rangle \) corresponding to \( f(x) = 1 \).

3.2. The general SAGE algorithm

The SAGE algorithm has proposed in [19]. It is an optimization of the EM algorithm [20]. It always converges faster than the EM algorithm. Its principle can be expressed as follows:

Provided that \( \theta \in \Theta \) is a set of parameters to be estimated from some observed data \( Y \in Y \) , and the observed data \( Y \) is the incomplete data. We can set \( Z \) as the complete data. There is many-to-one mapping \( Z \rightarrow Y(Z) \) from the complete data \( Z \) to the incomplete data or observed data \( Y \). The SAGE algorithm decomposes the observed (incomplete) data into its unobservable (complete) signal components and then estimates the parameters of each signal components separately. We can divide a set of parameters to be estimated \( \theta \in \Theta \) into different subsets. In each iteration, we only update one of the subsets \( \theta_m \) \((\theta_m = [\theta_m(1), \theta_m(2), \ldots, \theta_m(M)])\) and keep its complement \( \theta_{\bar{m}} \) fixed.

Provided that \( \hat{\theta}^{(i)} \) is the result of \((i-1)\)-th iteration. Hence at the \( i \)-th iteration, the Expectation (E) and Maximization (M) steps of the SAGE algorithm can be expressed as follow:

   **The E-step:**

   \[
   Q(\theta_m | \hat{\theta}^{(i)}) = E \left\{ \log p(Z | \theta_m, \theta_{\bar{m}}, y, \hat{\theta}^{(i)} ) \right\}
   \]

   **The M-step:**
\[
\theta_m^{[i+1]} = \arg \max_{\theta_m} Q(\theta_m | \theta_m^{(i)});
\]
\[
\theta_m^{[i+1]} = \theta_m^{(i)}.
\]

4. Proposed GS based data detection algorithm

4.1. The GS based data detection

The ML date detection for MIMO-OFDM systems is always used to compute the formula as follows
\[
\hat{x} = \arg \min_x \left\| y - H \cdot x \right\|
\]

Hence if we carry out it at the \(m\)-th receive antenna, we have to compute about \(2^N\) results \(\{x_1, x_2, \cdots, x_{2^N}\}\) corresponding to one subcarrier of OFDM symbol at \(N\) transmit antennas. And we have to search the minimum value of them. Its computational complexity is about \(O(2^N)\). If we use the GS based data detection to search the minimum value, it has a computational complexity \(O(\sqrt{2^N})\).

4.1.1. Databases preparation

First prepare a database with \(2^N\) elements which store all the transmit database, than we can compute all the value of (12). It will generate \(2^N\) results \(\{x_1, x_2, \cdots, x_{2^N}\}\) corresponding to one subcarrier of OFDM symbol at \(N\) transmit antennas. We store the results in the second database and make sure each \(x_i(i = 1, 2, \cdots, 2^N)\) corresponds to a quantum basis state \(|\psi_i\rangle\)(\(i = 1, 2, \cdots, 2^N\)).

Then we should find the minimum \(x_m = \min\{x_1, x_2, \cdots, x_{2^N}\}\) corresponding to a quantum basis state \(|\psi_m\rangle\).

4.1.2. GS based data detection

(1) Initialize the \(N\)-bit qregiste. We use the quantum basis state \(|\psi_i\rangle\)(\(i = 1, 2, \cdots, 2^N\)) in part 4.1.1 by Walsh-Hadamard transformation. Then we can get the superposition state of the qregiste:
\[
|x\rangle = \frac{1}{\sqrt{2^N}} \sum_{i=1}^{2^N} |\psi_i\rangle
\]

(2) Grover iteration \(G=UO\). We define a two thresholds function \(f(x)\) as follows:
\[
f(x) = \begin{cases} 
1, & \text{if } x = x_i \text{ and } x \leq x_j \\
0, & \text{otherwise}
\end{cases}
\]

Where the \(x_i, x_j\) are randomly chosen from the set \(\{x_1, x_2, \cdots, x_{2^N}\}\). We carry out the Grover iteration, which makes sure that we can get a solution with a high probability when taking measurement.
(3) Measurement. After $\frac{\pi}{4}\sqrt{2^N}$ times iteration, take a measurement on the qregiste. We can get the qregiste state $|\psi_m\rangle$ corresponding to $x_m$ in the second database. Then we can find the $k$th $X$ in the first database according to $x_m$.

4.2. SAGE Channel Estimation algorithm

4.2.1. Initial Estimation

We can provide a MAP estimator with the P length of training symbols as the initial estimation. The MAP estimator has been employed as one of the optimal channel estimators in iterative channel estimation and decoding of space time coded systems. So according to [21], the MAP estimator at the $m$-th receive antenna can be expressed as

$$\hat{H}_{m,\text{MAP}} = \begin{bmatrix} \hat{H}_{1,m}^{tr} & \cdots & \hat{H}_{N,m}^{tr} \end{bmatrix}^T$$

(15)

Where $R_{hh}^{tr} = E[H_{hh}^H (H_{hh})^H]$ is an expression of Power Delay Profile (PDP). $X_m^{tr} = [X_1^{tr}, X_2^{tr}, \ldots, X_N^{tr}]$ is the transmitted training symbols matrix, $Y_m^{tr} = [Y_1^{tr}, Y_2^{tr}, \ldots, Y_N^{tr}]^T$ is the received training symbols at the $m$-th receive antenna. $\sigma_n^2$ is the variance of AWGN noise.

According to (15), we can get the initial estimate of the transmitter frame by GS based data detection discussed in section 4.1.

4.2.2. The E-step

According to 3.2, given the complete data set $Z = \{Y, X\}$, $\theta \in H$ is the parameter vector to be estimated. The loglikelihood function of the parameter vector can be expressed as

$$\log p(Z \mid H) = \log p(Y \mid H, X) + \log p(X \mid H)$$

(16)

Because $\log p(X \mid H)$ is independent of $H$, it can be can be ignored [22].

At the $m$-th receive antenna, according to the Gaussian distribution, (16) can be expressed as

$$\log p(Y \mid H, X) = \sum_{i=1}^N \left[ (Y_m^i) - \sum_{n=1}^N X_n^i H_{n,m} - Y_m^i \sum_{n=1}^N (X_n^i H_{n,m})^H \left( \sum_{n=1}^N X_n^i H_{n,m} \right)^{-1} Y_m \right]$$

(17)

At the $i$-th iteration, In the E-step, according to (17) the (9) becomes

$$Q(H_{n,m} \mid H^{(i)}) = E\left[ \sum_{i=1}^I \log p(Y_m^i \mid H_{n,m}, H^{(i)}, X) \mid Y_m, H^{(i)} \right]$$

(18)

4.2.3. The M-step

In the M-step, we maximize the (19) by setting the gradient

$$\nabla_{(H_{n,m})} Q(H_{n,m} \mid H^{(i)}) = 0$$

(19)

According to (19), we can get the channel estimation $H_{n,m}^{(i+1)}$ between the $n$-th transmit antenna and the $m$-th receive antenna at the $(i+1)$-th iteration.
\[
H_{m}^{[\iota+1]} = \left[ \sum_{i=1}^{L} \left[ \hat{X}_{n}^{[\iota+1]} \right] \right] \cdot \sum_{i=1}^{L} \left[ \left( \hat{X}_{n}^{[\iota]} \right)^{H} Y_{m}^{k} \right] + \sum_{i=1}^{L} H_{m}^{[\iota]} X_{n}^{[\iota]}
\]

Where \( \hat{X}_{n} \) is calculated by GS based data detection with the channel estimate output \( H_{n,m}^{[\iota]} \) given by the \((\iota-1)\)-th SAGE iteration.

5. Application of Joint GS based data detection and SAGE based channel estimation algorithm

In this part, we propose a joint GS based data detection and SAGE based channel estimation algorithm.

We divide an \( L \) length transmit sub-frame of MIMO-OFDM system into some sub-blocks. In addition to the training symbols, it contains one OFDM symbol in every sub-block, and one OFDM symbol contains \( L \) sub-carriers. Then the proposed of joint GS based data detection and SAGE based channel estimation algorithm for one receive sub-frame at the \( m \)-th receive antenna as follows:

5.1. The first sub-block channel estimation and data detection part

Step 1: First we get the \( \hat{H}_{m}^{[\iota]} \) by (15) according to the training symbols in the head of \( L \) length sub-frame.

Step 2: We can detect the first sub-block transmit symbol by GS based data detection.

Step 3: According to the first sub-block symbols we got in step 2, we can get the channel estimation by (20).

Step 4: Then we update the first sub-block symbols by GS based data detection according to the result of channel estimate we got in step 3.

Step 5: Employ an iterative optimization loop which joints the step 3 and step 4 until converge.

Note: In this iterative optimization loop part, SAGE’s E&M steps in step 3 use the result of data detection we got in step 4.

Step 6: Output channel estimation and data detection of the last iteration in the progress of updating the first sub-block in step 5. So far we have updated the first sub-block.

5.2. The \((S+1)\)-th sub-block channel estimation and data detection part

Step 1: If convergence occurs in the \( S \)-th sub-block, the initial estimation of the \( k \)-th sub-carrier in the \((S+1)\)-th sub-block can be get by GS based data detection. Note that in this part of the GS based data detection algorithm, we use channel estimation of the last iteration in the progress of updating the \( S \)-th sub-block when convergence.

Step 2: Like the step 5 in 5.1, we employ an iterative optimization loop which joints the SAGE’s E&M steps and GS based data detection until converge.

Step 3: Output channel estimation and data detection of the last iteration in the progress of updating the \((S+1)\)-th sub-block in step 2. So far we have updated the \((S+1)\)-th sub-block.

Like the 5.2, by applying channel estimation of the previous sub-block transmit symbols to initial estimation in the current sub-block transmit symbols with GS based data detection, updating channel estimate and data detection until converge, we can get all of the sub-blocks’ the channel estimation and data detection. So far we have updated the transmit sub-frame at all transmit antennas. The joint algorithm is end.

6. Performance analysis

In this part, simulation is provided to show the performance of our proposed joint algorithm. We consider a 4×8 MIMO-OFDM system based on V-BLAST. The channel bandwidth of system is 20M.
The number of sub-carrier is 1024. 1 training symbol and 10 data symbols constitute a sub-frame. The prefix ratio is 27/128. The number of sub-frame is 10.

![Figure 2. BER performance comparison with BPSK](image)

**Figure 2.** BER performance comparison with BPSK

![Figure 3. BER performance comparison with QPSK](image)

**Figure 3.** BER performance comparison with QPSK

Fig. 2 and Fig. 3 show the BER versus SNR (E_b/N_0) at five schemes with BPSK and QPSK modulation, respectively. It is obvious that the proposed joint algorithm can nearly approximate to the ideal channel with ML data detection. It has better BER performance at both BPSK and QPSK than the ML date detection + LS channel estimation algorithm and ML date detection + LMMSE channel estimation algorithm. It is close to the optimal ML data detection + SAGE channel estimation, but with a complexity of \( O(\sqrt{2^N}) \) in data detection part better than ML data detection with a complexity of \( O(2^N) \).
Fig. 4 shows the BER performance with 10, 100, 400 length of transmit sub-frame, respectively. If we choose the long transmit sub-frame, although it can improve the bandwidth efficiency, the BER will increase. As the error in the channel estimation algorithm will progressively accumulate with the increase in length of sub-frame, hence we should choose an appropriate length of one transmit sub-frame according to the compromise of bandwidth efficiency and BER.

7. Conclusion

In this paper, we propose a joint GS based data detection and SAGE based channel estimation algorithm for MIMO-OFDM system. We use the Grover’s Quantum Search in data detection part to reduce the complexity, and employ an iterative optimization loop which joint the SAGE’s E&M steps and GS based data detection to update all the transmitted symbols. In the performance analysis part, we mainly consider the BER performance with different joint algorithm and different length of transmit sub-frame. Simulations results show that our joint algorithm have good BER performance which nearly approximate to the ideal channel with ML data detection, but with lower complexity. We also find that the BER increase with the increase in length of sub-frame, hence choose the right length is very important for the performance of joint algorithm.

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9. References


