Nondeterministic Vehicle Routing Problem: A Review

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Abstract

The purpose of this paper is to provide a comprehensive review of the nondeterministic Vehicle Routing Problem (NDVRP). In this paper, NDVRP is divided into two categories, singlet NDVRP and double NDVRP. Research progress of each category is reviewed, and the existing problems in contemporary study are summarized. Furthermore some issues for further research are also presented: build complex logistics system model with multiple uncertainty parameters, construct combination optimal model with inventory, location and other logistics activities, combine with E-commerce, the Internet of Things and other new technologies, improve existing intelligent heuristic algorithms or introduce new algorithm.

Keywords: VRP, Nondeterministic DVRP, Stochastic, Fuzzy, Fuzzy Random, Random Fuzzy

1. Introduction

Vehicle Routing Problem (VRP) is how to organize the optimal routing to make a fleet of vehicles to pass a series of loading and unloading depots orderly, meeting some constraints, such as the demand of goods, vehicle capacity constraint and time constraint, reaching some objectives of problems, such as, the shortest distance, the minimal cost and the least vehicle, etc. Since VRP was first proposed by Dantzig and Ramser in 1959 [1], it has attracted wide attention of experts, scholars and managers in different fields, and has become a study focus in operations research and logistics system optimization, growing up a multitude of research achievements.

In its initial stage, most traditional studies of VRP have assumed that all information is deterministic, including customer demand, vehicle, road condition, and decision-makers before constructing the routing. That is, traditional studies were deterministic VRP (DVRP). However, in actual logistics system, it is hard to describe the information of VRP as deterministic, due to factors such as the influence of weather, road conditions, vehicle speed, the individual needs of the customer, as well as the cognitive level of decision-makers and the uncertainty of these problems concentrates on the demand, time, etc. We called it nondeterministic VRP (NDVRP). Compared with the DVRP, NDVRP further reflects the time and demand changes, the impact of real-time vehicle routing assignment, which becomes more close to the actual production and logistics activities. However, the proposed algorithms and conclusions of DVRP were usually not applied to NDVRP. Moreover, due to the influence of some factors, such as time and demand, the mathematical description of NDVRP is more complex, and the difficulty of searching feasible solutions also increases. Therefore, the research on NDVRP has aroused more attention and become the focus of current VRP research[3].

This paper is organized as follows: in Section 2, the definition and classification of NDVRP are explained. In Section 3, 4, 5, the research status of SVRP, FVRP, FRVRP and RFVRP are introduced respectively. Then in Section 6, the existing problems of research are analyzed and the future research tendency is proposed. In the final section, we summarize this paper and point out some subjects for further research.

2. Definition and classification of NDVRP
NDVRP refers to the problem that the planner does not know all about the vehicle routing information before the start of the vehicle routing assignment, and some of the information may be uncertain, ambiguous, or even unknown. After the initial vehicle routing construction, the vehicle routing assignment may be changed [2]. Obviously, compared with DVRP, the NDVRP is a more general VRP.

In related researches, uncertainty includes 3 basic forms: randomness, fuzziness and roughness [3]. In some systems, the uncertainties above mentioned may coexist at the same time. At present, the uncertainty of VRP is mainly caused by randomness and fuzziness, and may use fuzzy random and random fuzzy to describe the coexisting phenomenon of randomness and fuzziness in VRP.

Since VRP is proposed, scholars have classified VRP according to the different standards and angles. With reference to Cao’s classification [4], and combine with the characteristics of NDVRP, NDVRP can be divided into singlet NDVRP and double NDVRP. The singlet NDVRP includes stochastic VRP (SVRP), fuzzy VRP (FVRP) and rough VRP. The double NDVRP includes fuzzy random VRP (FRVRP), random fuzzy VRP (RFVRP), fuzzy rough VRP, rough fuzzy VRP, random rough VRP, rough random VRP, dual fuzzy VRP, dual random VRP and dual rough VRP.

At present, studies of NDVRP mainly focus on SVRP and FVRP. FRVRP and RFVRP also begin to receive sparse attention in recent years, but these studies should be further improved. As far as we know, there is no research on rough VRP and other double NDVRP except FRVRP and RFVRP. Thus, this paper just reviews literatures in SVRP, FVRP, FRVRP and RFVRP fields.

3. The stochastic Vehicle Routing Problem (SVRP)

SVRP means the problem that some related information about the vehicle routing is not completely certain before the start of the vehicle routing assignment, the planner can just obtain statistical regularities of the information based on historical data or market survey. That is, some parameters in VRP are random, mainly including customer demand, the time of customer needs, customer distribution, vehicle that provides service, service time, travel time, and management level, etc. A good overview about SVRP is given in Gendreau and Laporte [5]. Currently, the study of SVRP major concentrates on the stochastic demand, stochastic travel time, and stochastic customer and so on.

3.1. VRP with stochastic demand (VRPSD)

Although the customer who needs service is determined, their exact demand is uncertain, and the planner can only obtain probability distribution of their demand. That is the problem of VRPSD. Three factors need to be considered in building VRPSD model: the scope of the service points that each circuit contains, the number of service failures allowed and strategy selection, mean value calculation of each circuit’s total distance. VRPSD is studied deeply and explored valuably through combining these three factors and has cost most study efforts. In the literature we collected, the first SVRP dates back to Tillman F in 1969 [6]. Tillman F studied VRPSD with multi-service center, and proposed heuristic saving algorithm to solve such problem based on C-W algorithm, provided that the vehicle empty driving or overloading should be punished.

Since then, scholars began to develop many new models and new algorithms for VRPSD. Early studies mainly used exact algorithm to solving such problem [7,8], such as branch and bound, integer L-shape, etc. Exact algorithms are able to find the optimal solution, but can not avoid the exponential explosion due to using rigorous mathematical methods, so much so tat these exact algorithms are only effective for solving small and medium VRP. With the deepening of the study, solving large-scale SVRP must rely on some kind of heuristics, such as particle swarm optimization, ant colony algorithm, genetic algorithm, simulated annealing algorithm, artificial neural network algorithm, etc. Each algorithm has its own limitations in solving practical problems. For example, global search capability of simulated annealing algorithm is insufficient, and particle swarm optimization is easy to fall into local optimal solution. Thus, the present study of such problems focuses on improving the heuristic algorithms or mixing using them [9-12].

3.2. VRP with stochastic travel time (VRPST)
Accurate vehicle travel time is uncertain due to the impact of factors such as road conditions, weather changes. The travel time between nodes is a random variable with some sort of statistical regularity. That is VRPST. Compared with VRPSD, the research of VRPST is less. Among them, the early typical representative is the study of Laporte [13], which assumed that travel time is random, established three models and used the branch and bound algorithm to solve the models. Based on the work of Laporte, Guo [14] provided a chance constrained model considering vehicle capacity constraints and solved the model by adopting genetic algorithm. Ichoua [15] presented a SVRP model depending on travel time based on the assumption of vehicle’s first-in, first-out and gave a parallel tabu search algorithm to solve this model. Taş [16] designed a random model considering soft time windows and stochastic travel time, and proposed a Tabu Search algorithm to solve VRPST.

3.3. VRP with stochastic customer (VRPSC)

Under the circumstance of VRPSC, customers appeared in a certain probability, but their needs are certain, and vehicle to provide service can not be overloaded. The vehicle must return to the depot if it carries maximum capacity. To date, VRPSC is rarely studied. Bertsimas [17] studied specifically the characteristics of random customer VRP, and designed heuristic methods of space-filling curve and probabilistic 2-opt edge interchange mechanism for solving this kind of problem. Laporte [18] employed integer L-shape method for solving stochastic customer VRP with the number up to 50. Bent[19] raised multi-depot stochastic customer VRP model, and constantly adjusted the vehicle routing planning according to the existing and future customers. In addition, Gendreau [8,20] and Xie[21] discussed VRP with both stochastic demand and stochastic customers.

Despite certain progress that the above mentioned SVRP has made in model building and algorithm, it has been almost limited to studying a single uncertainty caused by the random parameter. In logistics practice, there are many random factors coexisting. Therefore, in order to be closer to reality, comprehensively considered other uncertain factors, such as customers, road conditions and vehicle, has attracted attention of an increasing number of scholars. Astrid [22] and Tian [23] considered VRP with both stochastic travel time and stochastic service time; Hou [24] studied VRP with both stochastic demand and stochastic travel time, and so on.

4. The fuzzy Vehicle Routing Problem (FVRP)

Although information of SVRP is uncertain, there are some statistical rules to follow. However, FVRP arises whenever some elements of a given problem involve uncertainty and no statistical rules to follow. That is, some elements of the VRP have vagueness or ambiguity. For example, based on experience, a vehicle’s travel time can be described as "about 20 minutes", "between 20-40 minutes", and so on. Generally, such fuzzy elements can better reflect the reality. But it is very difficult to use algorithm of DVRP for solving FVRP, due to several fundamental properties of DVRP no longer hold in FVRP. Therefore, it needs to introduce new fuzzy programming models and algorithms to solve such problems. The general thought of solution is firstly to make information fuzziness, and then use fuzzy reasoning idea to build certain fuzzy criteria, finally convert the fuzzy variables into their crisp equivalents by using defuzzification. Currently, FVRP research mainly concentrates on the fuzzy demand, fuzzy due time and fuzzy travel time.

4.1. VRP with fuzzy demand (VRPFD)

VRPFD means that customers who need service are determined, but their exact demand is uncertain and the statistical rule of their demand is unable to obtain. There is no doubt that VRPFD is the most studied of FVRP field. As far as the best we know, the earliest description of VRPFD was offered by Teodorovic in 1996 [25]. Teodorovic studied VRPFD with one depot, developed the rules of fuzzy decision based on propensity score, proposed the first phase scan thought based on the sweep heuristic algorithm, and optimized vehicle routing after generating an initial solution. Since then, several scholars have used various heuristics to solve the VRPFD with some success [2, 26-32]. On the basis of fuzzy possibility [2,26-30], binary possibilities[31] and fuzzy credibility [32], they applied the improved heuristics algorithm or hybrid heuristics algorithm for solving VRPFD. The most widely
used algorithm is the improved or hybrid genetic algorithm [26-28]. In addition, other algorithms have also been applied to solve this kind of problems [2, 29-32], such as improved ant colony algorithm, hybrid differential evolution algorithm and improved particle swarm optimization.

4.2. VRP with fuzzy due time (VRPFDT)

VRPFDT is the problem that the time windows are replaced by fuzzy due time. Since the time windows in DVRP cannot reflect the real time preferences of customers. They may prefer to receive service at some moment in time windows. Their satisfaction may decline if the service is offered ahead of time or lagging behind. Hence the so-called VRPFDT emerged. Cheng [33] first proposed the notion of VRPFDT in 1995, established VRP model under the circumstance of single pick-up or single delivery, put forward improved hybrid genetic algorithm to solve such VRP model by employing the push-bump-throw process. After that, Teodorovic [34] researched on the fuzzy dynamic Dial-A-Ride problem, and further analyzed VRP with fuzzy travel time and fuzzy due time. Besides, other methods such as insertion heuristic algorithm, improved ant colony algorithm, genetic algorithm and so on play an important role in solving VRPFDT [35-37].

4.3. VRP with fuzzy travel time (VRPFT)

VRPFT is that in vehicle routing assignment, the travel time is fuzzy, while other parameters are given and deterministic. At present, the studies on VRPFT are relatively less with just several literatures. Teodorovic first introduced fuzzy theory into VRPFT in 1991 [38], supposed that travel times between nodes are fuzzy variables, constructed model of VRPFT and proposed Clarke-Wright algorithm for solving the model. On the basis of fuzzy possibility and credibility, Chen [39] and Zheng [40] studied VRPFT with time windows and put forward imperialist competitive algorithm and hybrid genetic algorithm. Jia [41] used genetic algorithm for solving VRPFT based on measurement method I₁. Zhang [42] and Brito [43] designed hybrid genetic algorithm for solving VRPFT based on fuzzy logic.

Besides, some literatures explored VRP with fuzzy cost coefficients [44], VRP with fuzzy time windows[45], VRP with both fuzzy travel time and fuzzy due time [34,46], and VRP with both fuzzy travel time and fuzzy service time [47], and so on.

In conclusion, the research of FVRP is mainly confined to some single fuzzy variable such as, fuzzy demand and fuzzy due time, etc. The study on considering comprehensively more fuzzy variables is less [34, 46, 47], especially that the role of distributor’s experience in VRP is rarely considered so that the practical applicability is weak.

5. Double nondeterministic Vehicle Routing Problem (Double NDVRP)

Double NDVRP is that randomness and fuzziness simultaneously appear in many cases in vehicle routing assignment. To describe this phenomenon, fuzzy random variable or random fuzzy variable is employed. Meanwhile, in vehicle routing assignment, it is possible that some existing elements are random, others fuzzy. Due to the fact that this kind of problem is random and fuzzy, it is difficult or even infeasible to use exact algorithm for solving and heuristics algorithm is hard to be applied directly. So the improved or new heuristics algorithm for solving is required.

5.1. The fuzzy random VRP (FRVRP)

Kwakernaak first proposed the concept of fuzzy random variable in 1978 [48], and defined it as a measurable function from a probability space to the set of fuzzy variables. In substance, fuzzy random variable is a random variable which has the value of fuzzy variable. In logistics practice, the historical data of probability distribution fitting certain variable usually can be obtained but are limited and not completely reliable. The value can be estimated by experts’ experience. Hence randomness and fuzziness simultaneously emerge and this variable can be described as fuzzy random one.

For example, if the market demand of a commodity in the next cycle is random, it may be strong with possibility of 0.5, or ordinary with possibility of 0.3, or lower with possibility of 0.2. The market
demand in each case can be estimated based on experience of managers, “if it is strong, demands will be 1000 units approximately”, “if it is ordinary, demands will be 600 units approximately”, “if it is lower, demands will be 300 units approximately”. That is, the demand of this commodity is fuzzy random variable. Before the vehicle routing is designed, if some elements are fuzzy random variables in VRP, such as demand, travel time, time windows, the so-called FRVRP comes into being. Currently, the literatures of FRVRP are less and start relatively late. As far as we know, Yang [49] introduced the fuzzy random variable into VRP research as early as 2008. Aiming at fuzzy random travel time and demand, Yang established the multi-objective fuzzy random chance-constrained programming model, assumed the fuzzy random travel time and fuzzy random demand is L-R fuzzy random variable, converted fuzzy random variable to a crisp equivalent model based on possibility and necessity measures and used hybrid multi-objective particle swarm optimization for solving. Malekly[50] studied capacitated VRP with fuzzy random demand and employed tabu search algorithm to solving this problem. Xu [51] studied VRP with fuzzy random time windows and used the fuzzy expected value to transform fuzzy information into a crisp equivalent and improved particle swarm optimization algorithm for solving.

5.2. The random fuzzy VRP (RFVRP)

Liu B [52] proposed the concept of random fuzzy variable, defining it as a random fuzzy variable is a function from a possibility space to the set of stochastic variables in 2002. In substance, the random fuzzy variable is a fuzzy variable which has the value of random variable. In logistics practice, distribution function of the random variable contains unknown parameters. For example, in the routing planning, demand follows certain determined distribution, but there is not enough sample data to obtain the accurate value of demand distribution parameter. Therefore, demand distribution parameter values need to be estimated subjectively according to experts’ experience. For instance, when demand follows a normal distribution with mean value about u, then the demand is called random fuzzy variable. When the vehicle routing is constructed, some elements in VRP are random fuzzy variables, FRVRP appears. He [53] first studied on FRVRP in 2005, supposed the demand is random fuzzy variable, on the basis of the maximum satisfying the capacity and arrival time, built VRP model to minimize the total travel time and converted it to \( \alpha \)-chance constrained programming model. He also obtained crisp equivalent model by introducing the set-valued function and used the improved genetic algorithm to solve FRVRP model. Shi [54] studied VRP with random fuzzy loss, built the max flow model based on credibility measure, and also used improved genetic algorithm to deal with the problem. Liu [55] established random fuzzy dependent-chance programming model with multi-type vehicle in fresh agriculture products distribution, designed the algorithm integrating random fuzzy simulation and genetic algorithm to solve the model. Overall, the researches on double NDVRP are in the initial stage. As the mathematical description of these problems is complex, solution methodologies of these problems are more intricate, related studies are less, among which, the team lead by Jiuping Xu, a professor in Sichuang University made comparatively early and deep researches [49, 51, 53, 55].

6. Review on the research of NDVRP

According to the literature review mentioned above, we summarized the current NDVRP research direction in table 1. As can be seen, studies of NDVRP have attracted more and more attention, but each research direction has different levels of development.

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Table 1. Summary of the current NDVRP research direction

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6.1. The existing problems of current research

At present, in the research field of NDVRP, there is certain gap between theoretical research and practical application, and comprehensive research needs to be strengthened. Research that combined with some new technologies such as E-commerce and the Internet of Things is seldom. The evaluation system of algorithm has not yet been built.

6.1.1. The distance between theoretical research and practical application: There is antinomy in the logistics system. For example, if the packing cost reduces, the insufficient protection of package may cause the increase of cost in loading and unloading, storing, and transportation. The enhanced service level can also bring about the rising logistics cost. Hence, in the decision-making process of VRP, the following situations should get more consideration, such as, multi-objective model, the combination of VRP with location-allocation and inventory. However, NDVRP researches mentioned above focus too much on single-objective rather than multi-objective and lack comprehensive studies which are combined with location and allocation, inventory and other logistics decisions. Besides, researches on open VRP with multi-type vehicle, multi-product and multi-depot are not enough and keep a distance with the practical application.

In logistics practice, there are many uncertainties which they even directly cross-impact. Most of the VRP researches focus on VRP with single uncertain elements, especially with uncertain demand or time, and lack comprehensive researches for multiple uncertain elements, such as study on comprehensively considering vehicle, customers, traffic, service levels, and other uncertainty. Meanwhile, researches on NDVRP are mainly confined to SVRP and FVRP, and those researches on double NDVRP such as FRVRP and RFVRP just start, without considering more complicated rough VRP and other double NDVRP, for example dual fuzzy VRP, dual random VRP, etc. So, they have a large gap with wide practical applications.

6.1.2. Less research on combining E-commerce and the Internet of Things: With the development of Internet, E-commerce has developed by leaps and bounds, which makes logistics demand rapidly grow, and put forward higher requirements to logistics service level and reaction speed. Consequently, it is very important and necessary to study VRP from distribution center to customers under E-commerce circumstance. Besides, the Internet of Things accelerates the process of logistics information, can make VRP more real-time and intelligent. Unfortunately, there is few NDVRP work combined with E-commerce, even no research of VRP combined with the Internet of Things at the moment.

6.1.3. Algorithm evaluation system requires to be established: Scholars often use heuristic algorithm to solve NDVRP, such as Sweep algorithm, C-W algorithm, K-opt algorithm, Two-stage algorithm, genetic algorithms, tabu search algorithm, ant colony algorithm, particle swarm optimization, etc. Now, it is very difficult to objectively assess these algorithms, because the selected test data of each algorithm are not all the same. We can only have empirical evaluation of the performance of NDVRP algorithm. Therefore, it is necessary to establish a unified algorithm evaluation system in order to objectively assess algorithm.

6.2. The future research tendency

In the future, research of NDVRP will center on building complex logistics system model with multiple uncertainty parameters, propose a better description of real NDVRP, especially design double NDVRP model. At the same time, multiple factors should be considered as much as possible, such as cost, time, and service to build multi-objective NDVRP model. The construction of VRP model with multi-type vehicle, multi-depot or multi-product is needed to come closer to reality. With the rapid development of E-commerce, the Internet of Things and other new technologies, study combined with them will become an important field of NDVRP. In addition, NDVRP combined with location and allocation, inventory and other logistics decision will attract much attention, such as, Inventory Routing Problem (IRP), Location Routing Problem (LRP), Combination of Location Routing and Inventory Problem (CLRIP).

The study of optimization methods will continuously introduce new intelligent heuristic algorithm, continue to improve the convergence of the algorithms, convergence rate, sensitivity analysis and
parameter design. Simultaneously, it will attract a great deal of attention to develop hybrid heuristic algorithm with a combination of different types of algorithms for the sake of complement each other. These hybrid heuristic algorithms are bound to play a more important role on the study of NDVRP. Moreover, to establish a unified algorithm evaluation system also has great practical significance in objectively assessing various algorithms.

7. Conclusion

NDVRP is one of the most important parts in the logistics system and a research area with bright prospect in which theory analysis and practice operation are closely related. As stated above, with the gradual deepening of study on NDVRP, the research development of singlet NDVRP is better, including SVRP and FVRP, but the study progress of double and multiple uncertainty is insufficient. Comprehensive study combined with location and allocation, inventory and other logistics decisions is not enough, and researches combined with E-commerce, the Internet of Things and other new technologies are inadequate. Moreover, new intelligent heuristic algorithm that has better performance should be developed and algorithm evaluation system has not been established yet, therefore present researches lack the ability to solve large-scale practical problems. Based on analyses, the real complex NDVRP should be constructed, and NDVRP optimization combined with location and allocation, inventory and other logistics decisions will be the emphasis and direction for future study.

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9. References