Portfolio Selection Model with the Measures of Information Entropy-Incremental Entropy-Skewness

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Abstract

In this paper, we develop a portfolio selection model with the measures of information entropy-incremental entropy-skewness (EESM) in which the risk of the portfolio is measured by information entropy, and the expected return is expressed by incremental entropy to indicate incremental speed of capital, and the risk of higher moment is measured by skewness. Then, we use the fuzzy programming technique to solve it. Finally, through a variety of empirical data sets from the Shanghai Stock Exchange in China, we evaluate the performance of the EESM in terms of several portfolio performance measures. The obtained results show that the EESM performs well relative to traditional portfolio selection models.

Keywords: Portfolio Selection, Information Entropy, Incremental Entropy, Skewness, Fuzzy Programming Technique

1. Introduction

Since the proposition of Markowitz’s famous mean-variance (MV) model, a great deal of studies have made a variety of significant contributions to improving the performance of the portfolio selection model. However, the MV model often leads portfolio to highly concentrate on only a few securities, which is contrary to the principle of diversification.

Recently, some scholars have applied the theory of entropy into the portfolio selection [1-9]. By utilizing a developed methodology of noise level estimation that makes use of properties of the coarse-grained entropy, Krzysztof and Holyst analyzed the noise level for the Dow Jones index and a few stocks from the New York Stock Exchange [1]. Jana et al. [2] first proposed a mean-variance-skewness model for portfolio selection and next added another entropy objective function to generate well-diversified asset portfolio within optimal asset allocation. Bera and Sung [3] provided an alternative approach by introducing cross-entropy measure which can automatically capture the degree of imprecision of input estimates, and Qin et al. [4] also discussed a cross-entropy minimization model in fuzzy environment. Huang [5] proposed two fuzzy mean-entropy models based on credibility and presented a hybrid intelligent algorithm. Rödder et al. [6] presented a new theory to determine the portfolio weights by a rule-based inference mechanism under both maximum entropy and minimum relative entropy. Xu et al. [7] investigated portfolio selection problems by utilizing the hybrid entropy to estimate the asset risk caused by both randomness and fuzziness. Zhang et al. [8] investigated a multi-period portfolio selection problem in fuzzy environment, and used the possibilistic semi-variance to appraise the portfolio risk, and the possibilistic entropy to measure the diversification degree of portfolio. A stochastic model of the investment portfolio based on maximization of entropy was proposed in Ref. [9]. In all, entropy has been increasingly applied to the portfolio selection as a...
measure of diversity because it is known that the greater the value of the entropy measure for portfolio weights, the higher the portfolio diversification is.

On the other hand, there exists not only the variance risk but also the skewness risk in the portfolio in Ref. [10, 11]. Moreover, the presence of negative skewness would increase the possibility of return on assets’ decline much larger than the one on assets’ rise. While the existence of excess peak will enable the likelihood of extreme events to increase greatly, both of which are known as the higher moment risk. These higher moment risks will affect the investment decisions of investors significantly, and great attention have already been given to the higher moment portfolio problems by some scholars. In particular, Prakash et al. [12], and Sunh and Yan [13] used the multi-objective programming method to deal with the optimization problem of three objectives: maximize the expected return, skewness and minimize the variance to select the portfolio with skewness risk respectively. These studies also showed that the skewness can enable the return of investors to be higher. Jondeau and Rockinger [14] used the expected utility function with Taylor expansions to discuss the problem of the asset allocation under non-normal conditions, and found that the risk of skewness and kurtosis have a significant effect on the financial investment decisions. Several models which involve the factor of skewness are proposed by some academics due to these facts above. For example, Bhattacharyya et al. [15] proposed a mean-variance-skewness portfolio model, and Kerstens et al. [16] compared this model with the mean-variance model based on the idea of shortage function. While, Usta and Kantar [17] added entropy measure into the mean-variance-skewness model to generate a well-diversified portfolio. However, in all the above papers, the mean returns of portfolio were measured by the arithmetic mean adopted by Markowitz which can not reflect the incremental speed of capital. Ou [18] used incremental entropy, one of the generalized entropies, to optimize portfolios and found the new portfolio theory based on incremental entropy carries on some aspects of Markowitz’s theory [18]. Indeed, the incremental speed of capital is a more objective criterion for assessing portfolio.

This paper presents a new portfolio selection model with the measure of information entropy-incremental entropy-skewness model (EESM). In this model, we replace arithmetic mean return with geometric mean one, and use the corresponding incremental entropy as a criterion for assessing a portfolio. Meanwhile, we use information entropy to predict the risk of portfolio and consider effects of skewness in the model. In order to reflect the subjective intention of investors in the portfolio more exactly and to enable the model to be more effective and feasible, we use the fuzzy programming technique to solve the model and built some empirical comparisons. The reminder of this paper is organized as follows: two classical portfolio selection models are presented briefly in Section 2. The EESM is introduced in Section 3. In Section 4 the empirical study is conducted to evaluate the performance of EESM with two classical portfolio selection Models. Finally, the conclusion is given in Section 5.

2. Two classical portfolio selection models

This section briefly presents Markowitz’s mean-variance model (MVM) and the mean-variance-skewness model (MVSM).

2.1. Mean-Variance Model (MVM)

In the MVM, mean expresses the expected return and variance measures the risk. Both of them are used to evaluate the portfolio’s value. The model is given as follows:

\[
\begin{align*}
\text{Min } & \quad X' \Sigma X \\
\text{s.t. } & \quad \sum_{i=1}^{N} x_i r_i \geq c \\
& \quad \sum_{i=1}^{N} x_i = 1, i = 1,2,..., N
\end{align*}
\]  

(1)

where the yield vector of \( N \) securities \( R = (r_1, r_2, \cdots, r_N)' \). The wealth fraction invested in the securities...
\(X = (x_1, x_2, \ldots, x_N)^T\), and the covariance matrix \(C\). Additionally, \(c\) represents the given expected return.

### 2.2. Mean-Variance-Skewness model (MVSM)

The model based on the mean, variance and skewness can be formulated as

\[
\begin{align*}
\text{Min } & X^T C X \\
\text{Max } & X^T S(X \otimes X) \\
\text{s.t. } & \sum_{j=1}^N x_j r_j \geq c \\
& \sum_{j=1}^N x_j = 1 \\
& x_j \geq 0, i = 1, 2, \ldots, N
\end{align*}
\]

(2)

where \(r\), \(X\), \(c\) and \(C\) are similar to the ones in model (1).

The second objective function in the model (2) can be expressed by

\[
X^T S(X \otimes X) = E[X^T R - E[X^T R]] = \sum_{i=1}^N \sum_{j=1}^N \sum_{k=1}^N x_i x_j x_k s_{ijk}
\]

(3)

where \(\otimes\) denotes for the Kronecker product and \(s_{ijk}\) represents the coskewness between the returns of asset \(i\), \(j\) and \(k\) for \(\forall (i, j, k) \in [1, 2, \ldots, N]\).

\[
s_{ijk} = E[(R_i - E[R_i])(R_j - E[R_j])(R_k - E[R_k])]
\]

(4)

### 3. Information entropy-incremental entropy-skewness model (EESM)

#### 3.1. Incremental Entropy

Incremental entropy is a kind of generalized entropy which can be defined as

\[
H = \sum_{k=1}^N p(x_k) \log \sum_{k=1}^N q_k R_k
\]

(5)

where assuming that the market consists of \(N\) securities and security \(k\) has \(n_k\) kinds of possible value for \(k = 1, 2, \ldots, N\). Therefore, the amount of all price combinations is \(W = n_1 \times n_2 \times \cdots \times n_N\), where \(i\)th one is \(x_i = (x_{i1}, x_{i2}, \ldots, x_{iN}), i = 1, 2, \ldots, W\); \(q_k, k = 1, 2, \ldots, N\) is the wealth fraction invested in security \(k\); \(R_k\) is the output ration of security \(k\) in \(i\)th price combination. In addition, \(R_{ik} = 1 + r_{ik}\) where \(r_{ik}\) represents the rate of return. We also assume that no short selling is allowed, which means \(q_k \geq 0\), \(k = 1, 2, \ldots, N\).

Additionally, the equation (5) can be converted to the formula as follows:

\[
H = \log \prod_{i=1}^N R_i^p(x_i)
\]

(6)

where \(R_i = \sum_{k=1}^N q_k R_k\).

We denote

\[
R_i' = \prod_{i=1}^N R_i^p(x_i)
\]

(7)

where \(R_i'\) is the geometric output ratio which can reflect the incremental speed of capitals, and the mean utilizes the arithmetic average output ratio to measure the earnings of portfolio. However, it is too proportional for \(q\) and to reflect the cumulative gain of fund. Therefore, the incremental entropy is a better measure than the mean to represent the investment income.
3.2. Minimum information entropy-maximum incremental entropy-maximum skewness model

In this section, we replace the variance of the MVM with the information entropy to measure the risk of portfolio, replace the mean with the incremental entropy to measure the expected return of portfolio, and also consider effects of skewness in the portfolio. The model is formulated as the following:

\[
\text{Min } S = -\sum_{i=1}^{W} p_i \ln p_i
\]

\[
\text{Max } H = \sum_{i=1}^{W} p_i \log \sum_{k=1}^{N} q_k R_{ik}
\]

\[
\text{Max } U = \sum_{k=1}^{N} \sum_{i=1}^{W} q_k \ln (q_k) s_{i,k,i,k}
\]

\[
\text{s.t. } \sum_{i=1}^{W} p_i = 1, p_i \geq 0, i = 1, 2, ..., W
\]

\[
\sum_{i=1}^{N} q_k = 1, q_k \geq 0, k = 1, 2, ..., N;
\]

\[
q_k \geq 0; k = 1, 2, ..., N; j = 1, 2, 3
\]

(8)

where \( S \) means the information entropy which is used to represent the risk of portfolio, \( H \) indicates the incremental entropy which can predict the portfolio’s return, \( U \) denotes the skewness, \( q_k \) is the wealth fraction invested in security \( k \), \( R_{ik} \) is the output ratio of security \( k \) in \( i \)th price combination, and \( s_{i,k,i,k} \) represents the coskewness between the returns of asset \( k \), \( k_1 \) and \( k_2 \). In addition, \( p_i \) is the probability of occurrence of \( i \)th price combination, but it can not be predicted. Obviously, the model is a multi-objective programming question. Therefore, the optimum proportion of portfolio under the conditions of minimum information entropy, maximum incremental entropy and maximum skewness can be obtained.

For the model (8), we can address it by the following steps:

Step 1, we assume that it is in a continuous period of time except holidays and vacations, \( r_k \) for \( k = 1, 2, \ldots, N \) represents the closing price of \( k \)th stock. Then, the output ratio security it can be defined as

\[
R_i = \frac{r_i - r_{i-1}}{r_{i-1}} + 1
\]

(9)

Step 2, according to the actual situations of market in China that there is a 10% limitation about fluctuations of increase and decrease of normal stocks’ prices per day, we can divide the interval of \([-10\%, 10\%]\) into 20 isotonic subintervals and denote \( i \)th subinterval as \( a_i \) for \( i = 1, 2, \ldots, 20 \).

Step 3, we can calculate \( n_{ki} \) which represents the frequency of the yield rate of \( k \)th stock on \( i \)th subinterval. Then, we define that if \( R_{ki} \leq -10\% \), the yield rate is on the subinterval \([-10\%, -9\%]\), and if \( R_{ki} \geq 10\% \), it is on the subinterval \([9\%, 10\%]\).

Step 4, we can calculate \( \rho_{ki} \) which represents the frequency rate of \( k \)th stock on the \( i \)th subinterval, and \( p_i = \frac{n_{ki}}{d} \). We can define that \( p_i \) is equal to \( \rho_{ki} \), this means that the frequency can be applied to reflect the probability distribution of the yield rate of single stock.

In order to adapt the synchronous analysis of multiple stocks, we define \( p_i \) as follows:

\[
p_i = \frac{\sum_{k=1}^{N} q_k p_k \lambda_k}{\left( \sum_{k=1}^{N} q_k \sum_{k=1}^{N} \lambda_k \right)}
\]

(10)

where \( i \) represents \( i \)th subinterval for \( i = 1, 2, \ldots, 20 \), \( k \) represents \( k \)th stock in the investment portfolio for \( k = 1, 2, \ldots, 20 \), \( p_{ki} \) represents the frequency of \( k \)th stock on \( i \)th subinterval, respectively, so
\( p_i = \frac{n_{i+1}}{d_i} \), and \( \lambda_k \) represents correlation coefficient of \( k \)th stock in the portfolio which is defined as follows:

\[
\lambda_k = \frac{\sum_{i=1}^{N-1} \mu_{ik+1}}{\sum_{i=1}^{N} \sum_{k=1}^{\infty} \mu_{ik+1}} \tag{11}
\]

where \( \mu_{ik+1} \) is the correlation coefficient between \( k \)th stock and \((k+1)\)th stock. As a result, the portfolio optimization model can be transformed as follows:

\[
\begin{align*}
\text{Min} \ S &= \sum_{i=1}^{N} q_{ik} \mu_{ik} \lambda_k \ln \left( \frac{\sum_{k=1}^{\infty} q_{ik} \mu_{ik} \lambda_k}{\sum_{k=1}^{\infty} q_{ik} \mu_{ik}} \right) \\
\text{Max} \ H &= \sum_{k=1}^{\infty} q_{ik} \mu_{ik} \lambda_k \\
\text{Max} \ U &= \sum_{k=1}^{\infty} q_{ik} \mu_{ik} \lambda_k \ln \left( \sum_{k=1}^{\infty} q_{ik} \mu_{ik} \lambda_k \right) \\
\end{align*}
\]

\[
\sum_{i=1}^{N} q_{ik} \mu_{ik} = 1, q_{ik} \geq 0, i = 1, 2, \ldots, W; k = 1, 2, \ldots, N
\]

\[
\sum_{k=1}^{\infty} q_{ik} \mu_{ik} = 1, q_{ik} \geq 0, k = 1, 2, \ldots, N
\]

s.t. \( q_{ik} \geq 0, k = 1, 2, \ldots, N; j = 1, 2, 3 \) \tag{12}

Moreover, in this model, the explanations of \( S, H, U, q_{ik}, R_{ik} \) and \( s_{ik} \) are similar to the ones in model (8).

### 3.3. Fuzzy programming technique to solve EESM

The model (12) is a multi-objective optimization question, and then we apply fuzzy programming technique to solve it. We denote \( R_p \) as income from investment, \( f \) as investment risk and \( u \) as skewness. We choose three fuzzy sets \( A_1, A_2, A_3 \) and the corresponding membership functions \( \mu_{A_1}, \mu_{A_2}, \mu_{A_3} \) from domain \( \{R_p, f, u\} \) to show the qualities of \( R_p, f \) and \( u \). We choose the membership function as follows:

\[
\mu_{A_1}(R_p) = \begin{cases} 
0, & R_p \leq R_p^\text{r} \\
\frac{R_p - R_p^\text{r}}{R_p^\text{u} - R_p^\text{r}}, & R_p^\text{r} \leq R_p \leq R_p^\text{u} \\
1, & R_p \geq R_p^\text{u}
\end{cases} \tag{13}
\]

\[
\mu_{A_2}(f) = \begin{cases} 
0, & f \leq f_\text{r} \\
\frac{f - f_\text{r}}{f_\text{u} - f_\text{r}}, & f_\text{r} \leq f \leq f_\text{u} \\
1, & f \geq f_\text{u}
\end{cases} \tag{14}
\]
where $f_-$ is the minimum value of risk, $f^-$ is the maximum value of risk, $\bar{R}_p$ is the maximum value of expectant income, $r$ is the required rate of return, $u_-$ is the minimum value of skewness and $u^-$ is the maximum value of skewness.

Therefore, we can transform the model (12) by using fuzzy programming technique to the one as follows:

\[
\begin{align*}
\max \mu \\
\text{s.t.} \quad H-r & \geq \mu \\
R_f-r & \\
f-S & \geq \mu \\
f-f^- & \\
U-u & \geq \mu \\
\end{align*}
\] (16)

As a result, it is easier to obtain the optimal solution of model (16) which is also the efficient solution of model (12).

4. The empirical study

In this section, we introduce the various portfolio performance measures and rolling window procedure to evaluate the performance of the EESM relative to the MVM and MVSM.

4.1. Portfolio Performance Measures

In order to evaluate the performance of portfolio models, a number of alternative performance measures have been proposed in the Ref. [19-22]. In this study, we consider some of these performance measures. First, there is no doubt that the Sharpe ratio (SR) is one of the most crucial classic metric indicators of portfolio utility. The Sharpe ratio is equal to the risk premium of the portfolio divided by the standard deviation, namely the formula as follows:

\[
SR = \frac{E(R_p) - R_f}{\sigma_p}
\] (17)

where $E(R_p)$ is the portfolio expected rate of return, $R_f$ is the risk-free rate, and $\sigma_p$ is the standard deviation of the portfolio. In addition, what should be claimed is that the higher the Sharpe ratio, the better the portfolio. However, since the SR is based on the mean-variance theory, it is only valid for normally distributed returns. Particularly, the SR can lead to misleading conclusions when the return distributions are skewed or present heavy tails. Several alternatives to the SR for optimal portfolio selection have been proposed in the literature. Some of these alternatives are presented as the following:

The adjusted skewness Sharpe ratio (ASR), which takes into accounts the skewness of portfolio, is defined as follows:
The mean absolute deviation ratio (MADR), which considers the risk as mean absolute deviation, is given as follows:

$$MADR = \frac{E(R_p)}{E|R_p - E(R_p)|}$$

(19)

The Sortino-Satchell ratio (SSR) and Farinelli and Tibiletti ratio (FTR), which are performance measures based on the partial moments and their formulas are given as follows, respectively:

$$SSR = \frac{E(R_p)}{\sqrt{E(\max(-R_p,0)^2)}}$$

(20)

$$FTR = \frac{\sqrt[\nu]{E(\max(R_p,0)^\nu)}}{\sqrt[\mu]{E(\max(-R_p,0)^\mu)}}$$

(21)

where $E(\max(-R_p,0)^2)$ is the lower partial moment of order 2. $E(\max(-R_p,0)^\nu)$ and $E(\max(R_p,0)^\nu)$ are the lower partial moment of order $\nu$ and the upper partial moment of order $\mu$, respectively. The selection of $\mu$ and $\nu$ are associated to investors' styles or preferences. We can consider the following cases for $\mu$ and $\nu$ according to: $\mu = 0.5, \nu = 2$ for a defensive investor; $\mu = 1.5, \nu = 2$ for a conservative investor; $\mu = 1, \nu = 1$ for a moderate investor. Additionally, it is known that if $\mu = 1, \nu = 1$, the FTR reduces to the Omega ratio.

4.2. Results of the Empirical Study

For comparing the performance of this model with other models, we select randomly two datasets of 40 stocks from Shanghai Stock Exchange in China, and the period of datasets span from January 4, 2010 to November 2, 2010 and from May 20, 2011 to March 21, 2012, respectively. The catalog of 40 stocks’ codes is shown in Table 1.

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Table 1 The catalog of the 40 stocks’ codes

First, we compute the portfolio weights according to the MVM, MVSM and EESM based on the sample estimates of the first dataset. Then, utilize these portfolio weight vectors $q_0$, the out-of-sample return of portfolio in the second dataset, denoted by $R_p'$, is calculated by $R_p' = q_0R_0'$, where $R_0'$ denotes the return vector in the second dataset. Thus, the out-of-sample returns generated by each of the considered portfolio models can be obtained. Based on the out-of-sample returns, the SR, ASR, MADR, SSR and FTR measures are calculated to evaluate the performance of the EESM relative to MVM and MVSM.

The values of mean, standard deviation, skewness, SR, ASR, MADR, SSR, FTR(0.5,2), FTR(1.5,2) and FTR(1,1) for each portfolio obtained from MVM, MVSM and EESM are respectively plotted in from Fig.1 to Fig. 10 in Appendix.

As seen from Fig. 1 to Fig. 10, the MVM provide poor results in terms of all performance measures
except FTR relative to the MVSM and EESM. Moreover, by contrast between indexes obtained from the EESM and the MVSM, it should be noted about Fig. 1, Fig. 3, Fig. 5, and Fig. 7 that EESM shows better performances in mean, skewness, ASR, and SSR than the MVSM. Additionally, it can be seen from the Fig. 2 that the fluctuation of standard deviation computed by the EESM is more stable as well except fewer extreme values. The results of SR and MADR in Fig. 4 and Fig. 6 respectively show that the values of both two performance measures above obtained by the EESM and MVSM are approximately similar, while EESM displays more steadily than the MVSM. On the other hand, in terms of the FTR (0.5, 2) and FTR (0.5, 2) and FTR (1, 1), we can find that there is no obvious differences among the EESM, MVSM, and MVM from Fig. 8-10. Overall, we can say that portfolios obtained from the EESM perform better in terms of variety portfolio performance measures than the MVM and MVSM.

5. Conclusions

We present a multi-objective model which includes the information entropy, the incremental entropy and the skewness. We use the incremental entropy to reflect the incremental speed of capital, and entropy to measure the risk of portfolio, and also consider effects of skewness. The fuzzy programming technique is applied to address the portfolio selection model. And then, by comparing their performance with the two classical models based on a series of advanced performance measures and a multitude of data sets from the Shanghai Stock Exchange in China, we find that the performance of EESM is better than the considered other models. The transaction costs in the EESM should be considered and applied into the financial market in the future.

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7. Appendix
Fig. 2. Standard deviation obtained from MVM, MVSM and EESM.

Fig. 3. Skewness obtained from MVM, MVSM and EESM.

Fig. 4. SR obtained from MVM, MVSM and EESM.
Fig. 5. ASR obtained from MVM, MVSM and EESM.

Fig. 6. MADR obtained from MVM, MVSM and EESM

Fig. 7. SSR obtained from MVM, MVSM and EESM
Fig. 8. FTR (0.5, 2) obtained from MVM, MVSM and EESM

Fig. 9. FTR (1.5, 2) obtained from MVM, MVSM and EESM

Fig. 10. FTR (1, 1) obtained from MVM, MVSM and EESM

8. References


