Analysis of Spatial Sampling Characteristics for the Circular Array

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Abstract

Uniform rectilinear array is commonly used to explain the correspondence relationship between spatial sampling and time sampling, while the characteristics of the spatial sampling is much unfamiliar than rectilinear array for the widely used circular array. In this paper, we pointed out that the circular array sampling sequence has the characteristics of the nonlinear frequency modulation (FM) and analyzed the optimal number of circular array combined with the multi-rate signal processing theory by the analysis on the characteristics of the circular array sampling.

Keywords: Circular Array, Equivalent Space-Time, Spatial Sampling, Nonlinear Frequency Modulation, Extraction

1. Introduction

The reception of space signals for the antenna array can be regarded as a sampling process on the signal. If the location of the starting sampling points was defined and the timeline direction was provided, the output of the array can constitute a discrete-time sequence. The most typical representative is the uniform rectilinear array. Based on the excellent correspondence between the spatial sampling of uniform rectilinear array and the corresponding frequency-time sampling, the correlated mature signal and system theory can be used to the analysis of spatial sampling [1-3].

In the actual array, circular array is commonly used for a greater surveillance area. In order to obtain better reception, the methods in increasing the pore size and in increasing the number of array elements are usually used to improve the reception performance for antenna array. However, the increase of pore size and the number of array elements is limited. Therefore, the number of array elements for the large antenna must be the limited number in an acceptable cost to the. Large shortwave arrays, for example, shows common uniform circular array with 40 elements, 48 elements, and 96 elements etc, and pore size ranging from 100 meter to 300 meter[4-5]. For the circular array, the issues about spatial sampling characteristics, the difference on spatial sampling with the rectilinear array, the relation between the reception effect and the pore size and the number of array elements are concerned for the antenna array design and array signal processing, which are based on the circular array. These issues must be answered through the theoretical analysis [6].

Through the analysis on the characteristics of the circular array sampling, we pointed out that the circular array sampling sequence has the characteristics of the nonlinear frequency modulation (FM). The demodulation results were also explained. Additionally, the optimal number of circular array was analyzed combined with the multi-rate signal processing theory.

The rest paper is organized as the following: In Section 2 introduce the concept of equivalent space-time. We explain the circular array and modulation in Section 3. Next, we explain the relationship of pulse compression and spatial spectrum in Section 4. In Section 5, we analyze the circular array y-rectilinear array transformation and circular array elements. Finally, we draw a conclusion in Section 7.

2. Equivalent space-time

Equivalence of airspace and time domain sampling usually described using uniform linear array [1].
In the process of time domain, the time signals were sampled for each \( t \cdot \) interval, frequency of time samples were \( f = \frac{1}{t} \). Then frequency domain was obtained using the discrete Fourier transform, which helps to get the information about signal spectrum. For a uniform linear array, single reception of each array element is equivalent to spatial discrete sampling of electromagnetic signals that have continuous distribution in airspace. The interval of space sampling is the array interval \( d \), and frequency is \( \frac{u}{d} = \frac{v}{d} \). Spatial spectrum formed in the conventional beam sampling process can be regarded as spatial Fourier transform. And the incident position of signal can be estimated based on the peak position of spatial spectral.

According to the hypothesis of the narrow band far field signal, time differences of receiving signal for any two array elements are:

\[
\tau_s = \frac{d \sin(\theta)}{\lambda} = du
\]

(1)

\( u \) is spatial frequency, and sampling sequence of \( M \) array element can be expressed as:

\[
A(\theta) = \left[ 1, e^{-j2\pi u d}, e^{-j2\pi u d}, \ldots, e^{-j2\pi u (M-1) d} \right]
\]

(2)

Similar with that interval of time sample should meet requirement of the Nyquist sampling theorem, intervals \( d \) of airspace sample must less than the half of space cycle of electromagnetic wave. That is:

\[
d \leq \frac{1}{2u} = \frac{\lambda}{2 \sin(\theta)}
\]

(3)

If no frequency aliasing within any point of scope was required, then the sampling interval of spatial should as follows:

\[
d \leq \frac{\lambda}{2}
\]

(4)

Corresponding relationships of equivalent space-time is listed in Table 1.

<table>
<thead>
<tr>
<th>Time</th>
<th>Time signal</th>
<th>Time sample</th>
<th>Frequency Discrete Fourier transform</th>
<th>Frequency spectrum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Space</td>
<td>Space incentive</td>
<td>Space sample</td>
<td>Spatial frequency (Direction)</td>
<td>Spatial discrete Fourier transform</td>
</tr>
</tbody>
</table>

What needed to be stressed is that resolution of time domain is proportional to the length of observation. As the samples points of time domain can be very huge, so it can be relatively easier to achieve high resolution. While sample points of spatial are determined by array element of space array, which are limited. And the resolution is proportional to the largest aperture of array that constitute by finite array element. Generally, the aperture of array is too small to successfully obtain a high space resolution. This is one difference among the airspace Fourier transform and time domain Fourier transform.

3. Circular array and modulation

Space sampling characteristics of circular array and line array have some kinds of differences. The number of array elements was defined as spatial sampling rate to facilitate the study, write as \( M \). 0 degrees is defined as starting time, and the time axis direction is clockwise, without considering the characteristics of the signal itself, and consider only the spatial phase shift the array. When sampling of electromagnetic waves on a circle with an aperture is \( 2R \) (\( R \) is radius) in space, the \( M \) sample points exactly can be regarded as a complete cycle, which is defined in the sequence of a cycle.

Electromagnetic information undergoes spatial sampling, Fourier transform is done for the sample sequences and the transformed result is defined as the spatial sampling spectrum. The spatial sampling characteristics of a circular array can be demonstrated in the frequency for some extent.
Taking the electromagnetic wave with \( \theta \) incident angle as sample, the \( i \) th sample point for spatial sampling of can be expressed as:

\[
a(i) = e^{-j2\pi / \lambda R \cos(2\pi i / M - \theta)}
\]

(5)

Assume the sequences \( A(\theta) \) composed by \( M \) sample points as following:

\[
A(\theta) = \left[ e^{-j2\pi / \lambda R \cos(\theta)} e^{-j2\pi / \lambda R \cos(2\pi M - \theta)} e^{-j2\pi / \lambda R \cos(4\pi M - \theta)} \cdots e^{-j2\pi / \lambda R \cos(2\pi - \theta)} \right]
\]

(6)

If \( M = 3600 \), the spectrum can be expressed in Figure 1 after the sequence with Fourier transformation.

![Figure 1. sequences spectrum for circular array sampling](image)

It is can be seen that the spectrum of the circular array sample sequence contains multiple frequency components. \( a(i) \) phase partial deducting, the following formal can be obtained:

\[
\Delta \phi(i) = \frac{-2\pi R(\cos(2\pi(i + 1) / M - \theta) - \cos(2\pi i) / M - \theta)}{\lambda} = \frac{-2\pi R(\cos(2\pi i / M - \theta)\cos(2\pi / M) - 1) + \sin(2\pi / M)\sin(2\pi i / M - \theta)}{\lambda}
\]

(7)

when \( M \to +\infty \),\( 2\pi / M \to 0 \), the \( \cos(2\pi / M) \to 1, \sin(2\pi / M) \to 2\pi / M \). Then

\[
\Delta \phi(i) = \frac{2\pi R2\pi \sin(\theta - 2\pi i / M)}{\lambda M}
\]

(8)

Because of \( f_s = M \), therefor the sample time interval is \( \Delta t = 1 / M \), and then the frequency for the time sequence of \( i \) th sample point can be expressed as:

\[
f(i) = \frac{\Delta \phi(i)}{2\pi \Delta t} = \frac{\Delta \phi(i) M}{2\pi} = \frac{2\pi R\sin(\theta - 2\pi i / M)}{\lambda}
\]

(9)

Form the above analysis, the space signal is modulated when it is received by circular array, resulting in a non-linear FM signals. The FM slope is sinusoidal change with a maximum
The frequency $f_{\text{max}} = \frac{2\pi R}{\lambda}$. The FM bandwidth for this signal and is independent of the sampling points, but only depends on the aperture wavelength ratio. For the time signal, the wider of spectral width the better time resolution; Similarly, for the spatial signal with modulation of the circular array, the wider of FM broadband the better azimuth resolution. Thus, for a circular array, a sufficiently large pore size is conducive to the accurate estimation on azimuth for the electromagnetic wave.

4. Pulse compression and spatial spectrum

Through the above analysis, we can confirm that the output signal of circular array is a non-linear FM signal. A similar is chirp signal (LFM), commonly used in the field of radar detection [2-4], can both take into account the long-range detection and high resolution. In the radar signal processing, acquisition of distance in formation can be realized through pulse compression of the LFM signal. Similarly, the pulse compression of a circular array output nonlinear FM signal can be used to access orientation information.

The matched filter of pulse compression can be expressed as:

$$H = \begin{bmatrix} e^{j2\pi/\lambda R \cos(2\pi/M)} & e^{j2\pi/\lambda R \cos(4\pi/M)} & \cdots & e^{j2\pi/\lambda R \cos(2\pi)} \end{bmatrix}$$

(10)

$$F = A(\theta) \otimes H = \text{IFFT}(\text{FFT}(A(\theta)) \bullet \text{FFT}(H))$$

(11)

Figure 2. The relations for two types signal processing procedure

Figure 3. Spatial spectrum of circular sampling sequence after pulse compression
Spatial spectrum obtained from the pulse compression have the same form with that obtained from the vector correlation method, conventional beam scanning method and other methods. Spatial resolution is limited by the aperture or said by the circular array modulation bandwidth, which is generally bad. So, modern spectrum estimation techniques are needed in order to obtain high spatial resolution.

5. Circular array-rectilinear array transformation

In equation (7), assume $X = FFT(A(\theta)) \bullet FFT(H(t))$, then $F = IFFT(X)$, which shows similar expression form with the spatial spectrum resulted from spatial sampling through Fourier transform for rectilinear array. If $X$ also has the line spectrum property of rectilinear array sampling results, the obtain process of $X$ can be regarded as the transformation process of the circular array to a linear array.

Assume $\overline{A}(K) = FFT(A(\theta))$, $\overline{H}(K) = FFT(H(\theta))$, then

$$X(K) = \overline{A}(K)\overline{H}(K)$$

(12)

For the kth point of $\overline{A}(K)$:

$$\overline{A}(k) = \sum_{i=0}^{M-1} e^{-j2\pi/kR \cos(\theta - 2\pi i/M)} e^{-j2\pi k M \theta}$$

(13)

According to Fourier-Bessel principle, when $M > 4\pi R / \lambda$:

$$\overline{A}(k) \approx M j^{-k} J_k(-2\pi R / \lambda) e^{-j\theta}$$

(14)

$J_k(x)$ is the first type Bessel function. The frequency points in band are considerate.

$k \in [-2\pi R / \lambda, 2\pi R / \lambda] \cap Z$, $[x]$ is integer $X$, $Z$ is integer set.

Similarly:

$$\overline{H}(k) \approx M (-1)^k j^{-k} J_k(-2\pi R / \lambda)$$

(15)

Then:

$$X(k) = M^2 J_k^2(-2\pi R / \lambda) e^{-j\theta}$$

(16)

It is seen that the phase of $X$ varies linearly with the changes of the sample point $k$, indicating that $X$ corresponds one single-frequency signal. However, the magnitude of each sample point is not consistent, but is changing in $M^2 J_k^2(-2\pi R / \lambda)$. Given the magnitude of each sample point is changeable. Following treatment can be accepted: transformation through a circular array of arrays sampling results into a virtual line array sampling windowed.

Each sample point multiplies $1 / M^2 J_k^2(-2\pi R / \lambda)$, this transformation generates a uniform rectilinear array in a circular array patterns space [5-6]. The virtual rectilinear array converted from circular array remains the azimuth resolution isotropic property of the circular array, which is different with the real rectilinear array; compared with the true rectilinear array with a same aperture, the spatial resolution of virtual rectilinear array converted from a circular array is poorer than the maximum resolution of the true rectilinear array, the relative properties can refer the corresponding literature [7]. Some algorithms that only suitable for uniform rectilinear array would also be used in a circular array through the transformation of circular array - rectilinear array [8-10].
6. Circular array elements

From the spectrum of circular array sampling sequence as $M = 3600$, we can see that oversampling phenomenon is obvious. Therefore, we can extract form the sample sequence to reduce the sampling rate, that is to say to reduce the number of array elements. According to multi-rate signal processing theory, in order to protect the signal is not distorted after extract for the signal with sampling rate $M$, its maximum extract interval is $\frac{M}{M_B} \Delta B$, the signal bandwidth is $\Delta B = 2f_{\text{max}}$. When $M = 3600$, $\frac{R}{\lambda} = 5$, the number of array elements required is:

$$N = \frac{M}{\Delta M} = \Delta B = 2f_{\text{max}} = \frac{4\pi R}{\lambda} = 62.84$$

(17)

In this case, the required number of array elements is 63. In the actual case, the number of array element for most of the arrays cannot guarantee to meet the lossless extraction conditions cover the whole band. Taking the common short wave big basic direction finding array with 40 elements and aperture 100 m ($R = 50m$) as example, it is recognized as a new array extracted from a array with 3600 elements. In order to make the spatial sampling sequence does not appear aliasing phenomenon in the frequency after the Fourier transform, it is need to satisfy the conditions for the signal wavelength:

$$\lambda \geq \frac{4\pi R}{N}$$

which requires the signal frequency $f \leq \frac{C}{\lambda} = \frac{NC}{4\pi R}$, $C$ is the speed of light, from the formula (8). For aperture 100 m and 40 elements array, $f < 19MHz$. The spatial sampling sequence spectrum of 40 elements array with 3600 elements array is compared as following:
Figure 5. The spectrum comparison before and after extraction for spatial sampling of circular array (without aliasing)

Figure 6. The spectrum comparison before and after extraction for spatial sampling of circular array (Close to without aliasing)
Because of the required frequency is less than 19MHz, this is hard to realize distortion spatial sampling in the full-band signal for the shortwave. The influence of spectrum aliasing at high the frequency induced by the extraction process will inevitably lead to the sampling results are subject to certain "pollution". Taking synthetic beam as example, this effect is manifested in the form of elevated side lobe.

Conventional large basic direction finding station with Wullenweber antenna array, the auditory measurement is generally composed only 12 elements antenna, the corresponding angle range is ±08°.
degrees, which is equivalent to small the FM bandwidth to cover the higher frequency band. Assume the number of original array elements $M$ is 3600, array element number in 108 degree range is 1080, the FM output sequence bandwidth of spatial sampling is:

$$\Delta B = 2f_{\text{max}} = \frac{4\pi R \sin(2\pi i / M)}{\lambda} = \frac{4\pi R f_i \sin(2\pi i / M)}{C} = \frac{4\pi R f_i \sin(2\pi * 540 / 3600)}{C}$$

(18)

Where, $C$ is the speed of light. Consider the extract interval is $\Delta M = \frac{M}{N}$, $\Delta B \leq \frac{f_s}{\Delta M}$ is need to avoid the occurrence of frequency aliasing. Because $f_s = M$, so the maximum frequency for the optimal covering frequency range is:

$$f_{\text{max}}^{\text{opt}} = \frac{N}{\Delta B} = \frac{40 * 3e8}{4\pi * 50 * \sin(2\pi * 540 / 1800)} = 23.6 MHz$$

(19)

Certainly, because of the transportation of short-wave is in the ionosphere, usually need a certain angle of incidence, which is equivalent to smaller $\frac{R}{\lambda}$. In this case, $f_{\text{max}}^{\text{opt}}$ will increase. The addition of reflective network for the antenna array can also reduce the elevated sidelobe caused by aliasing and to effectively improve the coverage at higher frequency.

7. Conclusion

In this paper, spatial sampling results of circular array were analyzed by the relative digital signal processing theory. The analysis methods might help the engineering staff to have a better understanding on spatial sampling characteristics of circular array. Additionally, this work could also provide the reference for performance analysis of circular array, large circular array lineup, simulation beam synthesis. Finally, this paper opened useful information for spatial sampling characteristics analysis of other arrays.

8. References