A Novel High Spectral Efficiency Time Domain Convolutional Multiplexing Technique

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Abstract
This paper proposes a high spectral efficiency signal transmission technique, using parallel concatenated structure and employing inter-symbol interference (ISI) to get code constraint, which is called Time Domain Convolutional Multiplexing (TDCM) technique. The novel technique does not extend the system bandwidth, whose quantization levels are increased linearly with multiplexing number. TDCM technique draws on the merits of Turbo code and employs the idea of parallel concatenated structure, which can be proved by simulation and analysis that it outperforms high order modulation system, as well as TDCM with single path transmission scheme under the same data rate. The superiority of TDCM technique is even more obvious when forward error correction (FEC) encoder with high correction capability is employed at the transmitter.

Keywords: Time Domain Convolutional Multiplexing, High Spectral Efficiency, Power Spectral Density, Inter-Symbol Interference, Maximum Likelihood Sequence Detection

1. Introduction

With the development of wireless communication and the surging mobile data traffics, the requirements for anytime, anywhere communications are getting higher and higher. As the bandwidth resources are very limited and valuable, there is a trend for applying high data rate and high spectral efficiency techniques in the wireless communication system design.

Traditionally, there are some techniques to improve the spectral efficiency, such as high order quadrature amplitude modulation (QAM) and orthogonal frequency division multiplexing (OFDM) technique. However, there are some drawbacks, the signal to noise ratio (SNR) threshold for QAM technique is high, and the quantization levels are increased exponentially with modulation order, which is a great challenge for system power efficiency.

The spectral efficiency of OFDM technique is theoretically much higher than conventional frequency division multiplexing technique, but the overhead of cyclic prefix and guard band can’t be ignored, as well as the problem of high peak to average power ratio, thereby the improvement of spectral efficiency is very limited.

TDCM employs overlapped time domain waveforms to multiplex the signals, thus the symbol period is shortened. In the same time interval, the transmitted symbol waveforms are increased, which improves the data rate. TDCM is turned out to have lower SNR threshold than orthogonal multiplexing methods like QAM under the same spectral efficiency; even with low order modulation, such as binary phase shift keying (BPSK), high spectral efficiency can be obtained by increasing the number of overlapped waveforms.

Similar to TDCM technique which employs ISI, there are partial response signaling and generalized faster than Nyquist signaling (FTN), whose symbol intervals are designed artificially to lower than Nyquist interval. The similarity is that certain performance gains can be obtained by introducing ISI, for example, partial response signaling can be used to shape the spectrum of transmitted signals.

There are also some differences. The ISI of partial response system is under controlled, and the theoretical maximum spectral efficiency is 1 baud/Hz. The design propose of FTN is to maintain the free distance of sequences while the symbol interval is shortened. TDCM intends to improve the distribution of code weight by introducing ISI, so as to get certain encoding gain, as well as to realize data transmission with high data rate and high spectral efficiency.
The proposed technique combines parallel concatenated structure of Turbo code with time domain convolutional multiplexing scheme, whose SNR threshold and quantization levels are lower than QAM technique. Besides, the TDCM technique would not expand system bandwidth from the theoretical derivation as shown in the following section. The system spectral efficiency is increased linearly with the number of overlapped waveforms, while the detecting complexity of optimal algorithm is exponential with the equivalent constraint length, thus the number of overlapped waveforms is practically limited.

The paper is organized as follows. In section 2, system structure and models are introduced, as well as the derivation of power spectral density (PSD) for TDCM. In section 3, detailed parameters and detecting algorithm are depicted. In section 4, numerical results are shown in additive white Gaussian noise (AWGN) channel. In section 5, conclusion remarks and future work are summarized.

2. System Description

2.1. Basic principles and structure diagram

The transmission model of present technique is shown in Figure 1. Assuming that the information source are composed by memoryless and equiprobable bit streams, the information bits are mapping into BPSK symbols, which is \( X = [x_i, x_{i+1}, \ldots, x_N] \) with the \( n \)-th transmitted symbol \( x_n \in \{+1, -1\} \), \( 1 \leq n \leq N \), \( N \) is the frame length. \( g(t) \) denotes the impulse response of pulse shaping filter, whose time duration is \( T \), and there is \( g(t) = 0 \) when \( t < 0 \) or \( t > T \).

After constellation mapping, the data symbols are processed through serial to parallel block, and the 2\(^{nd} \) to the \( L \)-th parallel branches are delayed in turn with \( \Delta T, 2\Delta T, \ldots, (L-1)\Delta T \), where \( \Delta T = T/L \). All the delayed signals are summed and transmitted through channel, corrupted by Gaussian white noise \( n(t) \). The single side PSD of noise is \( N_0 \). The received signals can be expressed as:

\[
y(t) = \sum_{n=1}^{N} x_n g(t - n\Delta T) + n(t) = s(t) + n(t)
\]  

When the sampling intervals are integer times of \( \Delta T \), and \( [g_0, g_1, \ldots, g_{L-1}] \) are the \( L \) samples of impulse response \( g(t) \), \( k = 1, 2, \ldots, N + L - 1 \), the sampled discrete signals are depicted as:

\[
y_k = \sum_{n=1}^{N} x_n g_{k-n} + n_k = \sum_{i=0}^{L-1} x_{k-i} g_i + n_k
\]  

Obviously, the received signals are the convolution of transmitted symbol sequence and \( L \) samples of impulse response for pulse shaped filter. In the time duration \( T \), \( L \) symbol waveforms are overlapped together, which constitute the constraint relations among adjacent symbols. The basic TDCM transmission structure can be modeled as a tap delay line model with code rate 1 and constraint length \( L \), which is shown in Figure 2.

![Transmission model of TDCM technique](image.png)

**Figure 1.** Transmission model of TDCM technique

![Equivalent tap delay line model of TDCM technique](image.png)

**Figure 2.** Equivalent tap delay line model of TDCM technique
The differences between TDCM and traditional convolution code (CC) are depicted as follows. Firstly, the taps of Figure 2 are decided by the samples of shaping filter impulse response; secondly, the input of TDCM can be symbols after constellation mapping, such as BPSK modulation, the input symbols $x_n \in \{+1, -1\}$, while, the input of CC are elements in binary finite field, and $x_n \in \{0, 1\}$; thirdly, when the length of data frame is far more longer than $L$, the code rate of TDCM is about 1, while the code rate of CC is lower than 1, thus the data signaling rate per unit time (i.e. the spectral efficiency) of TDCM is far more higher than traditional CC.

Figure 3 shows the system model of traditional single path transmission TDCM system (a), as well as the parallel concatenated TDCM system (b). For the traditional TDCM system, low order modulator such as BPSK are used, and TDCM detecting is implemented with maximum likelihood sequence detection (MLSD), where the detecting complexity is exponential increased with $L$. For the parallel concatenated TDCM system, the I path and Q path TDCM signals are multiplexed with the complex field form "I + j Q", then corrupted by Gaussian white noise. At the receiver side, the I/Q path separation block is carried out, and then iterative detecting of parallel branches is implemented. The detail iterative detecting method will be depicted in later section.

2.2. Power spectrum density of output signals

For the convenience of analysis, it’s supposed that $g(t)$ is impulse response (i.e. gate function) of rectangular wave with time duration $T$, which is normalized time waveform of transmitted symbols. It can be expressed as:

$$g(t) \begin{cases} 1/T, & t \in [0, T), \\ 0, & t \not\in [0, T). \end{cases} \quad \int_0^T |g(t)|^2 dt = 1$$

(3)

Then the transmitted signals $s(t)$ of TDCM can be denoted as:

$$s(t) = \sum_{n=0}^{N-1} x_n g(t - nT / L)$$

(4)

It’s assumed that the data sequences are independent zero-mean normalized complex sequences, which satisfy:

$$\frac{1}{2} E[s_n s_n^*] = \delta_{n,n}, \quad E[s_n] = 0$$

(5)

In order to get the PSD of TDCM signals, the complex envelopes of the signals’ autocorrelation function are calculated firstly:

$$R(\tau) = \frac{1}{2N} E\left[ \sum_{n=0}^{N-1} s_n g(t - nT / L) g(t - nT / L - \tau) \right]$$

$$= \frac{1}{2N} \sum_{n=0}^{N-1} x_n g(t - nT / L) \sum_{\tau=0}^{L-1} g(t - nT / L - \tau) \delta_{n,n}$$

(6)

Obviously, it only relates to the transmitted symbol waveform $g(t)$. When $g(t)$ depicts gate function of rectangular wave, $R(\tau)$ can be calculated as the form of triangular waveform, which is:

$$R(\tau) = \delta_{\tau,T} \left[ 1 - \frac{\tau}{T} \right], \quad |\tau| \leq T$$

$$R(\tau) = 0, \quad |\tau| > T$$

(7)
According to [8], the complex envelope of PSD is the Fourier transform of autocorrelation function:
\[ R(\tau) \leftrightarrow G(f) = \text{Sinc}^2(\pi f T) \cdot \delta \] (8)

It turns out that the overlapped signals transmission method of TDCM has no influence on the PSD of transmitted signals. PSD of TDCM only relates to the specific waveforms of \( g(t) \), as the time shifts of TDCM signals only affect the phase characteristics, and PSD has nothing to do with phase characteristics of transmitted signals.

### 2.3. System spectral efficiency and quantization levels

According to Nyquist criterion [7], the minimum two sided bandwidth of shape forming filter to ensure there are no ISI between samples is \( 1/\Delta T \), when the time interval of transmitted symbols is \( \Delta T \). The symbols are often mapped into high-order modulated constellations for high spectrum efficiency. Assuming that the modulation order is \( M \), each symbol carries \( \log_2 M \) bits data, the maximum spectral efficiency of ISI free system is:
\[ \eta_m = \log_2 \frac{M}{1/\Delta T} = \log_2 M \text{(bps / Hz)} \] (9)

From the derivation of above section, there is the conclusion that the signal transmission method of TDCM does not expand the system bandwidth, and the shaped pulses of TDCM system are not limited by Nyquist criterion. It’s assumed that the double side bandwidth of certain shaped pulse is \( W \), the time bandwidth product of \( g(t) \) is \( WT \), thus the system spectral efficiency of TDCM system can be expressed as Equation (10).
\[ \eta_m = \frac{N \cdot \log_2 M}{W (N + L - 1) \Delta T} = \frac{N \cdot \log_2 M}{W (N + L - 1) / L} = \frac{L \cdot \log_2 M}{WT} \text{(bps / Hz)} \quad [N >> L] \] (10)

It can be seen that if the shaped pulse filter is given, the system spectral efficiency of TDCM system is increased linearly with \( L \), which is the multiplexing number of TDCM (i.e. the number of overlapped symbols). Thus the total data rate and the spectral efficiency can be highly increased when the parameter \( L \) of TDCM technique is increased.

The quantization levels of multi-levels modulation are increased exponentially with the modulation order, while that of TDCM technique are increased linearly with \( L \). The power efficiency of TDCM technique is higher than traditional high order modulation technique. It’s supposed that the modulator of TDCM is BPSK, and the shaped pulses are rectangular waves.

The quantization levels of TDCM are \( y \in \{ \pm 2, 0 \} \) with the number being 3 when \( L = 2 \), while the quantization number of QPSK under the same efficiency is 4; the quantization levels of TDCM are \( y \in \{ \pm 3, \pm 1 \} \) with the number 4 when \( L = 3 \), while the quantization number of 8PSK is 8; the quantization levels of TDCM are \( y \in \{ \pm 4, \pm 2, 0 \} \) with the number 5 when \( L = 4 \), while the quantization number of 16QAM is 16; the quantization levels of TDCM are \( y \in \{ \pm 5, \pm 3, \pm 1 \} \) with the number 6 when \( L = 5 \), while the quantization number of 32QAM is 32.

### 3. Detailed parameter design

#### 3.1. Interleaver

In the parallel concatenated TDCM system, the data correlation of parallel branches is reduced even eliminated through the effect of interleaver, which plays an important role in the iterative decoding block. The interleave depth and interleaver structure are the concerned parameters of TDCM system, which have certain influence on the system performance.

In [9-10], there are conclusions that in the high SNR region, the performances of parallel concatenated codes are decided mostly by the free distances of codeword distance spectrum, while the free distances are affected mainly by the low weight input sequences. The structure of interleaver has direct influence on the output weight of low weight input sequence, thus it is the important factor to influence the codeword free distance.
The structure of interleaver is the major factor for the system performance in the high SNR region, while in the low SNR region, interleave depth is the main factor. In the paper, several kinds of interleavers are introduced [11], such as the array interleaver, where data symbols are written by row and read by column, random interleavers, whose interleave patterns are generated randomly by computer, and S interleavers, where the S parameter is the minimum interval of adjacent jump distance. Besides, the different interleave depths are also considered in the system design.

3.2. FEC

It’s verified that the iterative gains of uncoded parallel concatenated TDCM system are limited after the third iterative detecting. For the further improvement on the system performance, the FEC block is introduced in the system, which would decrease the SNR thresholds greatly.

![Figure 4. (a) The encoder structure of LTE-Turbo code; (b) The encoder structure of TPC](image)

Turbo code has been chosen as the channel coding method for physical layer technical standard [12] of long term evolution (LTE) systems, which is also applied in the present system. The encoder structure of LTE-Turbo code with parallel concatenated convolutional code are shown in Figure 4(a), whose component codes are two eight state convolutional codes and code rate is 1/3 without puncture. The dash lines are applied for trellis termination. The transfer function of the component codes is depicted as:

\[ G(D) = \left[ \frac{g_1(D)}{g_2(D)} \right] = \left[ \frac{1 + D + D^2}{1 + D + D^2} \right] \]

(11)

Turbo product code (TPC) is another alternative FEC code for the TDCM system, the encoder of which is shown in Figure 4(b). TPC codes [13] are composed by information bits, column parity check bits, row parity check bits and parity check bits’ parity check bits. There are two linear block codes, called \( C(n_1, k_1, d_{min}) \) and \( C(n_2, k_2, d_{min}) \), and the former code is used to encode the information bits with \( k_2 \) rows into row parity check bits with \( k_2 \) rows and \( n_1 - k_1 \) columns, then the later code is used to encode the bits of \( n_1 \) columns, include information bits and row parity check bits, into column parity check bits with \( n_2 - k_2 \) rows and \( k_1 \) columns based on the column information bits, as well as the column parity check bits based on the row parity check bits with \( n_2 - k_2 \) rows and \( n_1 - k_1 \) columns.

3.3. Detection algorithm

Traditionally, the MLSD of ISI systems are implemented with Forney receiver [14] or Ungerboeck receiver [14]. It has been proved that [14] the received sequences \( y(t) \) are the sufficient statistics of transmitted symbol sequences \( x(t) \), thus the MLSD can be carried out with \( y(t) \). For the single path TDCM system, the MLSD is Viterbi algorithm in practice.

For the parallel concatenated TDCM system, the soft information of two parallel branches needs to be iterated, thus BCJR algorithm [16] is applied in the receiver side. There is log likelihood ratio (LLR) information output for each symbol when certain systems use BCJR algorithm to detect the data, and the systems can obtain the minimum symbol error ratio. The iterative decoding procedure is shown in Figure 5. When the BCJR algorithm is used in the TDCM system, there are some modifications in the detecting algorithm.
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**Figure 5.** Iterative detect structure diagram for parallel concatenated TDCM

Assumed that the frame size is \( N \), the number of overlapped symbols is \( L \), and BPSK modulator is used, the state at time \( n-1 \) and time \( n \) of certain path in Trellis diagram are respectively \( M_{n-1} \) and \( M_n \), and the input data symbol of the corresponding transfer branch is \( x_n \), the output symbol through TDCM block is \( s_n \), the posterior probability of \( x_n \) based on received sequence \( y \) can be expressed as:

\[
P(x_n = \pm 1 | y) = \frac{P(x_n = \pm 1, y)}{P(y)}
\]  
(12)

\( X^+ \) and \( X^- \) are defined as the sets of branches respectively for input symbols \( x_n = 1 \) and \( x_n = -1 \), \( y_{n}^{x_n} \) depicts the received sequence \( \{y_1, y_2, \cdots, y_{n-1}\} \), \( y_{n+1}^{x_n} \) depicts the received sequence \( \{y_{n+1}, y_{n+2}, \cdots, y_{n+l}\} \). According to Bayesian formula and the Markov property of ISI model, the joint probability \( P(y) \) can be calculated as:

\[
P(x_1 = 1, y) = \sum_{x_{1:n}} P(M_{1:n}, x_1 = 1, y) = \sum_{x_{1:n}} P(M_{1:n}, y_{1:n}, y_{1:n+1})
\]  
(13)

According to reference [15], there are definitions as follows:

\[
\alpha_{x_n}(M_{n-1}) = P(M_{n-1}, y_{n-1})
\]  
(14)

\[
\beta_{x_n}(M_n) = P(y_{n+1} | M_n)
\]  
(15)

\[
y_n(M_{n-1}, M_n) = P(y_n | M_{n-1}) P(y_n | M_n) = P(x_n) P(y_n | x_n)
\]  
(16)

The equation (13) can be rewritten as:

\[
P(x_1 = 1, y) = \sum_{(M_{1:n}, x_1, y_1)} \alpha_{x_n}(M_{n-1}) y_n(M_{n-1}, y_{n-1}) \beta_{x_n}(M_n)
\]  
(17)

In equation (16), \( P(x_n) \) represents a priori probability of transmitted data symbols, \( P(y_n / x_n) \) depicts the channel transfer probability function. For AWGN channel, there is the conclusion:

\[
P(y_n | x_n) = \frac{1}{2\pi\sigma^2} \exp \left[ -\frac{(y_n - x_n)^2}{2\sigma^2} \right]
\]  
(18)

It's assumed that the initial state and end state are all-zero state for the paths in Trellis diagram of TDCM signals. The algorithm parameters can be initialized as:

\[
\alpha_{x_n}(M_n) = \begin{cases} 1, & M_n = M_{\text{zero}} \\ 0, & M_n \neq M_{\text{zero}} \end{cases}
\]  
(19)

\[
\beta_{x_n^{-1}}(M_{n-1}) = \begin{cases} 1, & M_{n-1} = M_{\text{zero}} \\ 0, & M_{n-1} \neq M_{\text{zero}} \end{cases}
\]  
(20)

Then the equation (17) can be rewritten from equations (18-20), and the LLR of posterior probability for each input symbol can be calculated as:

\[
\text{LLR}_p(x_n) = \ln \frac{P(x_n = +1 | y)}{P(x_n = -1 | y)} = \ln \sum_{x_{1:n-1}} \alpha_{x_n}(M_{n-1}) y_n(M_{n-1}, y_{n-1}) \beta_{x_n}(M_n)
\]  
(21)

Assuming that the information source data are composed by equiprobable bits, the a priori probability LLR for each input symbol is initially:

\[
\text{LLR}_a(x_n) = \ln \frac{P(x_n = +1)}{P(x_n = -1)} = \ln \frac{1/2}{1/2} = 0
\]  
(22)
Thus the output extrinsic information would be the posterior LLR minus a priori LLR, and the whole iteration procedure are processed as shown in Figure 5.

4. Numerical results

The simulation results are based on the Monte Carlo method, with the assumption that the channel state information is known both by the transmitter and receiver. The bit error ratio (BER) performances in Eb/N0 (i.e. EbN0 for simplicity) under different scenarios are shown as follows, where Eb depicts the bit energy and N0 represents single side PSD of white noise.

Table 1. System parameters and simulation condition

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frame size</td>
<td>1000/4096/65536</td>
<td>FEC</td>
<td>LTE-Turbo code and TPC</td>
</tr>
<tr>
<td>Frame number</td>
<td>10000</td>
<td>Interleave depth</td>
<td>4096/65536</td>
</tr>
<tr>
<td>L of TDCM</td>
<td>5,8</td>
<td>Interleave structure</td>
<td>Array/Random/S interleavers</td>
</tr>
<tr>
<td>High order modulator</td>
<td>32QAM, 256QAM</td>
<td>Detecting algorithm</td>
<td>Viterbi and BCJR algorithm</td>
</tr>
</tbody>
</table>

In Figure 6, the TDCM system performances with L=5, are compared with 32QAM under the same spectral efficiency. The performance gain in Eb/N0 for the single path transmission scheme is about 2.5dB under the BER of 3*10^-5, while the iterative gain for the parallel concatenated TDCM after the 3rd iteration is about 6dB under the same BER ratio. Obviously, the parallel concatenated structure can improve the system performance greatly without loss of spectral efficiency.

In Figure 7, the performance influence of interleave depth for array interleavers are testified. It’s can be seen that when the interleave depth is increased, there is notable performance improvement for the TDCM system, which is about 2.5dB after the 3rd iteration.

In Figure 8, the performance influence of different interleaver structures are testified. It’s shown that the performances of 1st iteration are about the same, the S interleaver and random interleaver can obtain similar performance after 3rd iteration, and the performance of array interleaver is slightly worse than the other interleavers in the TDCM system.

In Figure 9, the TDCM system performances with L=5 and TPC (64, 57) whose code rate is 0.79, are compared with 32QAM with TPC (64, 57) code under the same spectral efficiency. After the 4th iteration, TDCM with parallel concatenated structure can obtain about 1dB in Eb/N0 compared with TDCM with single path transmission scheme, and the performance gain between TDCM and high order modulation system is about 2dB in Eb/N0.

In Figure 10, the TDCM system performances with L=8 and TPC (64, 57) whose code rate is 0.79, are compared with 256QAM with TPC (64, 57) code as the FEC under the same spectral efficiency. After the 4th iteration, TDCM with parallel concatenated structure can obtain about 1dB in Eb/N0 compared with TDCM with single path transmission scheme, and the performance gain between TDCM and high order modulation system is about 8dB in Eb/N0. The superiority of TDCM technique is even bigger when the overlapped symbols number is increased compared with high order modulation technique.

In Figure 11, the TDCM system performance with L=5 and TPC (26, 32) whose code rate is about 0.66, are compared with LTE-Turbo code whose code rate is 2/3. After 1st iteration, the later can obtain about 1dB performance gain than the former scheme, and the performance gap of the former are shortened to about 0.2 dB after 4th iteration.
Figure 6. Performance comparison of TDCM with different structures

Figure 7. Performance comparison of TDCM with different interleave depths

Figure 8. Performance comparison of TDCM with different interleaver structures

Figure 9. Performance comparison of TDCM with FEC (TPC (64, 57), L=5)
5. Conclusion

A high data rate and high spectral efficiency signal transmission technique, using parallel concatenated structure and overlapped waveforms transmission scheme, which employing inter symbol interference to get code constraints called TDCM technique is proposed. TDCM technique does not expand the system bandwidth, and the quantization levels of which are increased linearly with multiplexing number, which shows better power efficiency than high order QAM modulation system. The system performance of TDCM technique outperforms high order modulation system, as well as TDCM of single path transmission scheme under the same data rate. For a certain scenario, the performance gain can be 8dB in Eb/N0. When FEC encoder with high correction capability is employed in the transmitter, the superiority of TDCM is more obvious and lower Eb/N0 thresholds can be achieved. The proposed channel coding scheme can be used in cooperative communication system combined with network coding just as reference [17] shown, which will be the future work for consideration.

6. References


