Research on Bit and Power Allocation for MIMO Relay System

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Abstract

Bits and power allocation is an important issue of adaptive resource allocation in MIMO-OFDM system. In this paper, an improved algorithm, bits and power allocation algorithm based on water-filling theorem, is proposed and used to the single-user MIMO amplify-and-forward relay system. The algorithm which adopts the method of bisection to search water-filling line can reduce the number of iterations with low complexity compared with the traditional water-filling algorithm. Simulation results show that system capacity and bit error rate performance can be improved effectively.

Keywords: MIMO Relay System, Water-Filling Theorem, Bits And Power Allocation

1. Introduction

In the next generation (B3G/4G) of wireless mobile communication system, high rate and multi-business information transmission are the most outstanding characteristics. MIMO (Multiple Input Multiple Output) and OFDM (Orthogonal Frequency Division Multiplexing) technology as the key of next generation mobile communication system, the combination can effectively combat multipath fading and improve the information channel capacity and bit rate [1]-[2]. While the introduction of relay technology, not only can expand the cell coverage area, but also effectively resist channel fading, overcome the near-far effect [3]. Therefore, relay technology was introduced into MIMO-OFDM system, can effectively use the advantages of these technologies to further improve system performance.

For single-user MIMO-OFDM systems, adaptive resource allocation mainly are allocation of transmission bits and transmit power adaptively in the sub-carriers, so as to effectively improve the transmission performance of system. In the current study of adaptive resource allocation, mainly for MIMO-OFDM system, rarely consider joining relay. The main algorithm of bits and power allocation is water-filling algorithm [4], greedy algorithm [5], PSO (Particle Swarm Optimization) algorithm [6], etc. In addition, due to the use of different standards, different scholars have also made different researches. In reference [7], the adaptive sub-channel and power joint allocation was studied in MIMO-OFDM system. A low complexity degree of joint sub-channel and power allocation algorithm was proposed to expand coverage and improve the channel reliability. [8] proposed a SVD (singular value decomposition) assisted transmission power allocation scheme with unknown all channel state information. The algorithm can effectively improve the BER performance and reduce the residual interference. [9] in order to maximize the SLNR (Signal to Leakage Noise Ratio), a power allocation algorithm was proposed to maximize total system capacity with iterative algorithm that can effectively improve system performance and maximize the SLNR.

In this paper, a MIMO-OFDM AF (Amplify and Forward) relay system is discussed in the total transmit power under certain conditions. By allocating bits and power on all sub-carriers, allowing the system to meet the maximum system capacity and reduce BER (Bit Error Rate) requirements.

2. System model

The system under consideration is a single-user MIMO-OFDM AF relay system as shown in figure 1. the sender and the receiver has $N_s$ and $N_r$ antennas respectively, the relay is a AF relay with $N_r$ antennas, $H_1$ and $H_2$ respectively denotes the channel gain matrix between the sender to the relay and the relay to the receiver. Assuming channel state information is completely known, the system uses AF relay with TDD (Time Division Duplex) mode, namely that in the first time slot, the sender adaptively allocates transmitted bits and transmit power based on the instantaneous gain of each sub-carrier, and
put the distribution information feedback to the receiver, then send to the relay through the antenna after the space-time transformation and OFDM modulation. In second time slots, the relay receives the signal and amplifies forward to the receiver, so the receiver can demodulate the received signal base on the bits allocation information.

\[
\begin{align*}
\begin{array}{c}
\text{Bit allocation information} \\
\end{array}
\end{align*}
\]

Figure 1. MIMO-OFDM AF relay system model

For MIMO system, MIMO channel can be equivalent to several parallel SISO (Single Input Single Output) sub-channels, each carrying much OFDM sub-carrier information. Assuming the sender sends the signal as

\[
x = [x_1, x_2, \cdots, x_N]^T
\]

(1)

Where \( x_i, i = 1, 2, \cdots, N_s \) denote the send signal on the \( i \)th antenna of sender. If the channel gain matrix \( H_1 \) and \( H_2 \) are known in the system, which are expressed as

\[
H_1 = \begin{bmatrix}
h_{11}^1 & \cdots & h_{1N_s}^1 \\
\vdots & \ddots & \vdots \\
h_{N_s1}^1 & \cdots & h_{N_sN_s}^1
\end{bmatrix},
H_2 = \begin{bmatrix}
h_{11}^2 & \cdots & h_{1N_s}^2 \\
\vdots & \ddots & \vdots \\
h_{N_s1}^2 & \cdots & h_{N_sN_s}^2
\end{bmatrix}
\]

(2)

Then, the signals received by relay antenna array can be expressed as

\[
R = H_1 x + n_1
\]

(3)

While \( n_1 \) is an AWGN (Additive White Gaussian Noise) matrix from the sender to the relay.

Then the relay amplifies the receiving signal and forwards to the receiver, so the receiver receives the signal as

\[
y = H_2 (GR) + n_2
\]

\[
y = H_2 G H_1 x + H_2 G n_1 + n_2
\]

(4)

Which, \( G \) is the amplify factor by AF relay, \( n_2 \) is an AWGN matrix from the relay to the receiver, \( H_1 \) is the equivalent channel gain, \( n_1 \) is the system noise. It can be obtained by the SVD of \( H_1, (i = 1, 2) \):

\[
H_i = U_i \Lambda_i V_i^H, \quad i = 1, 2
\]

(5)

Which \( U_i \) and \( V_i \) \((i = 1, 2)\) both are unitary matrices, \( A_i \) is diagonal matrix,

\[
\Lambda_1 = \text{diag}\left[\sqrt{\alpha_1}, \sqrt{\alpha_2}, \cdots, \sqrt{\alpha_{\text{rank}(U_i)}}\right], \quad \text{and} \quad \alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_{\text{rank}(U_i)} \geq 0,
\]

\[
\Lambda_2 = \text{diag}\left[\sqrt{\beta_1}, \sqrt{\beta_2}, \cdots, \sqrt{\beta_{\text{rank}(U_i)}}\right], \quad \text{and} \quad \beta_1 \geq \beta_2 \geq \cdots \geq \beta_{\text{rank}(U_i)} \geq 0.
\]

So the output signal \( y \) is

\[
y = U_2 \Lambda_2 V_2^H G U_1 \Lambda_1 V_1^H x
\]

\[
+ U_2 \Lambda_2 V_2^H G n_1 + n_2
\]

(6)
So, $y$ can be regarded as the system divided into $K$ parallel SISO sub-channel output, and $K = \min \{\text{rank}(H_1), \text{rank}(H_2)\}$. Thus, MIMO sub-channels that simplify to $K$ parallel SISO sub-channels, amplify factor $G$ can directly influence the equivalent channel gain.

3. Bits and power allocation based on water-filling theorem

Water-filling algorithm which can effectively improve the system capacity is a classical algorithm for power allocation. But it has high complexity and its complex process for searching water-filling line. A bits and power allocation algorithm based on water-filling theorem is proposed, which adopts the method of bisection to search water-filling line and simplifies the solving process. At the same time, times of iterations are not necessary. The method can greatly reduce the computation complexity.

Assuming the number of sub-carriers in system is $N$. The receiver estimates the gain of each sub-carrier after transmitted by relays is $\lambda_n (n=1,\cdots,N)$. $b_n$ and $P_n$ are the bits and power which allocated on the $n$th sub-carrier respectively. The relationship of them is show as follows

$$b_n = \log_2 \left( 1 + \frac{\lambda_n P_n}{\Gamma \sigma^2} \right)$$  \hspace{1cm} (7)

or

$$P_n = \frac{\Gamma \sigma^2}{\lambda_n} \left( 2^{b_n} - 1 \right)$$  \hspace{1cm} (8)

$\Gamma$ is SNR in a certain BER, $\sigma^2$ is noise variance of AWGN. For M-QAM modulation, $\Gamma$ and BER $P_e$ have the relationship without consideration of channel coding as

$$\Gamma = -\ln(5P_e)/1.5$$  \hspace{1cm} (9)

Based on water-filling theorem, $P_n$ is given by

$$P_n = \max(0, \mu - \Gamma \sigma^2 \lambda_n^{-1})$$  \hspace{1cm} (10)

The purpose of the algorithm is to maximize the system capacity under the strict of the conditions of $\sum_{n=1}^{N} P_n \leq P_T$.

Get the bounds of water-filling line first to solve water-filling line

$$\mu_{\min} = \frac{1}{N} \left( \frac{P_T + \Gamma \sigma^2 N}{\lambda_n} \right)$$

$$\mu_{\max} = \frac{1}{N} \left( \frac{P_T + \Gamma \sigma^2 \sum_{n=1}^{N} \frac{1}{\lambda_n}}{\lambda_n} \right)$$  \hspace{1cm} (11)

While $n' = \arg \max_{n=1\cdots N} \lambda_n$, $b_n$ is rounded down as the number of transmitted bits is an integer. Then the bounds of water-filling line is

$$\hat{\mu}_{\min} = \frac{1}{N} \left( \frac{P_T + \Gamma \sigma^2 N}{\lambda_{n'}} \right)$$

$$\hat{\mu}_{\min} = \mu_{\min} + \max \left\{ \frac{1}{\lambda_n} \left( 2^{b_n} - 2^{b_{n'}} \right), n = 1, \cdots, N \right\}$$

$$= \frac{1}{N} \left( \frac{P_T + \Gamma \sigma^2 \sum_{n=1}^{N} \frac{1}{\lambda_n}}{\lambda_n} \right) + \max \left\{ \frac{1}{\lambda_n} \left( 2^{b_n} - 2^{b_{n'}} \right), n = 1, \cdots, N \right\}$$  \hspace{1cm} (12)

The flow diagram of bits and power allocation based on water-filling theorem is shown in figure 2.
Figure 2. The flow diagram of bits and power allocation based on water-filling theorem

In the figure, \( b_i \) and \( P_i \) are the results of the number of bits and power allocation in every sub-carriers. In the actual MIMO-OFDM system, if the number of sub-carriers is \( N \), MIMO sub-channel is equivalent to multiple parallel SISO sub-channels, each sub-channel has \( N \) sub-carriers.

4. Performance simulation analysis

Assume that the number of sub-carriers is 64, the total number of bits sent is limited to 128, the average power on each sub-carrier is 1W, the variance of AWGN is 0.01, the bandwidth is 1bps and the system error is 0.01. M-QAM modulation is used, BER is \( 10^{-4} \). Several times of simulations are
taken to prove stability of the system since the channel gain is randomly generated Rayleigh channel. The result of only one time simulation is shown in Figure 3, 4 and 5.

The system sender, relay, receiver has 2 antenna, through SVD, MIMO channel is decomposed into two independent SISO sub-channels in parallel way, so a total of 128 carriers. In figure 3, the first 64 carriers such as the red solid line is the 64 sub-carriers formed by first equivalent parallel SISO sub-channel, the last 64 carriers such as the blue dotted line is the second equivalent parallel SISO sub-channel formed on the 64 sub-carriers. It can be seen that in the different sub-carriers, according to the different channel gain, the required transmission bits and transmit power are different. A good carrier in the channel response gets more bits and poor response to the channel carrier little or no allocation of bits. For transmitting power, a good carrier in the channel response on the distribution of transmit power less and the carrier with poor channel response on the distribution of more transmit power, which meets the water-filling theorem.

Figure 4 shows the comparison of the system capacity. The equal power allocation, water-filling allocation and the bits and power allocation are compared in the figure. It is shown that the system capacity increases as the SNR increases. But little change in equal power allocation, and the bits and power allocation based on water-filling theorem significantly higher than others in the capacity.

Figure 5 shows the comparison of the three algorithms in system BER. It is seen that the system BER decrease as SNR increases. But compared to equal power allocation, although water-filling algorithm can improve the system capacity, it is higher than equal power allocation on the BER. The algorithm of bits and power allocation based on water-filling theorem is better than others on BER performance. Compared to equal power allocation, BER of $10^{-3}$ in the vicinity, bits and power allocation algorithm makes the system SNR about 3dB performance improvement.

![Figure 3. The result of bits and power allocation](image-url)
Simulation results given above show that the proposed algorithm is better on transmission performance compared with water-filling algorithm and equal power allocation algorithm. It can increase system capacity and also reducing the system BER.
5. Conclusion

In this paper, the problem of adaptive resource allocation for maximizing the system capacity is discussed in a single-user MIMO AF relay system. With water-filling algorithm can effectively improve the system capacity, it applied in MIMO relay system to adaptively allocate resource, a bits and power allocation algorithm based on water-filling theorem is proposed, which can reduce the computing complexity compared with conventional water-filling algorithm. In addition, the proposed algorithm can effectively improve system capacity, reduce system bit error rate, enhance the transmission performance of system, to make MIMO relay system transfer information more quickly and efficiently.

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7. References