Carrier Frequency Offset Estimation Based on Unitary-ESPRIT for Interleaved OFDMA Uplink Systems

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Abstract

A novel carrier frequency offset (CFO) estimation algorithm (Unitary-ESPRIT algorithm) is proposed for orthogonal frequency-division multiple-access (OFDMA) systems. The proposed algorithm is formulated in terms of the real-valued computations throughout except the data preprocessing. Compared with other recently proposed CFO estimation approaches, Unitary-ESPRIT algorithm offers a number of advantages. Firstly, the presented approach reduces the computational complexity significantly by dealing with only real-valued computations. Secondly, Unitary-ESPRIT incorporates the data stacking technology and the unitary transformation, which leads to an improved estimation performance. Simulation results illustrate the efficacy of the proposed algorithm.

Keywords: Orthogonal Frequency-Division Multiple-Access (OFDMA), Carrier Frequency Offset (CFO), Unitary Transformation

1. Introduction

In an orthogonal frequency-division multiple access system, multiple users are permitted to transmit data simultaneously through distinct sub-channels which are composed of the certain orthogonal subcarriers [1,2]. Besides possessing low-complexity modulation technique and high spectrum efficiency, OFDMA provides two further advantages:

(1) OFDMA can take advantage of frequency diversity by means of distributed subcarriers for a single user. Distributing a user's subcarriers pseudorandomly throughout the whole band means that some of the user's subcarriers likely would not experience fading while some of the user's other subcarriers likely would.

(2) OFDMA can take advantage of multiuser diversity through contiguous subcarriers. Multiuser diversity occurs because different users at different locations would likely experience different channel responses, thus the system can improve a particular user's link by assigning to that user a set of contiguous subcarriers which experience the best channel condition.

Therefore, OFDMA has been or is being considered for various wireless communication systems, such as IEEE 802.16 [3], 802.20 [4], 802.22 [5].

However, similar to OFDM, OFDMA is extremely sensitive to the carrier frequency offsets (CFOs) caused by oscillator mismatch and Doppler shift. The uncompensated CFOs will result in loss of orthogonality among subcarriers and introduce intercarrier interference (ICI) as well as multiple access interference (MAI), thus leading to severe performance degradation. Compare with the OFDMA downlink system, the CFO estimation in the OFDMA uplink system is critically important, since different users may have different CFOs, and all CFOs have to be simultaneously estimated by using the received signal which is the superposition of signals from all active users.

Recently, many CFO estimation schemes for uplink OFDMA systems have been proposed [6]-[13]. In [6], the CFOs are estimated by maximizing the mean likelihood function via exploiting an important sampling technique. In [7], an effective iterative scheme, which is referred to as the alternating-projection frequency estimator (APFE), is proposed. However, it requires an exhaustive grid searching.
to estimate each CFO which results in unattractive complexity. To reduce the complexity, the estimator in [8] replaces the matrix inversion in APFE with an approximated matrix extension. However, it requires a large number of subcarriers. By exploiting the repetitive structure of the training sequences, the alternative low complexity estimator [9] is proposed. The estimator in [10] takes advantage of constant amplitude zero autocorrelation (CAZAC) training sequences to obtain the CFOs. However, it causes an irreducible error floor. In [11], a simple iterative algorithm is derived via exploiting the fact that the tile structure in 802.16e provides inherent MAI power compression in the frequency domain. However, the estimators [6]-[11] are based on training sequences.

When training sequences are not available, the CFO for each user cannot easily be obtained. Fortunately, this problem can be overcome with a proper carrier assignment scheme (CAS), such as interleaved CAS. Furthermore, the interleaved CAS, where the subcarriers allocated to each user are equi-spaced in the whole frequency band, can maximize the frequency diversity and increase the capacity of OFDMA systems. With the interleave-based CAS, the CFO estimators proposed in [12]-[15] exploit the inherent periodic structure of each user’s signal. Due to the spectrum peak searching involved, the computational complexity of the MUSIC estimator [12] is not attractive. The ESPRIT [13] estimator can avoid the complex peak searching at the expense of the loss of estimation performance. The CFO estimator [14] is proposed by defining a CFO matrix. In [15], an optimum CFO estimator is proposed by truncating the series expansion of the correlation matrix in maximum likelihood function.

It should be notice that most of CFO estimators are based on the complex computation with huge computational load. In order to reduce the computational complexity but gain an improved performance, the Unitary-MUSIC estimator [16] based on the real-valued computation is proposed via exploiting the unitary transformation. However, it still needs to deal with peak searching. In this paper, a CFO estimation algorithm, named as Unitary-ESPRIT algorithm, is proposed for the interleaved OFDMA uplink systems. Compared with the previous works, the proposed algorithm provides several attractive features. Firstly, the proposed algorithm is derived in terms of the real-valued computations throughout except the data preprocessing, which reduces the computational complexity considerably, since the associated eigenvalue decomposition is for the real-valued matrices whose size is the same as the complex-valued matrices. Secondly, the proposed algorithm provides an improved CFO estimation accuracy by exploiting the data stacking technology and the unitary transformation.

The rest of this paper is organized as follows. Section II introduces the interleaved OFDMA uplink systems model. The proposed method is derived in Section III. Section IV discusses the computational complexity of the proposed algorithm and other relevant methods. Simulation results are given in Section V. Section VI presents the conclusion.

2. Data model for OFDMA uplink

Consider the interleaved uplink of an OFDMA system with \(N\) subcarriers in which \(K\) active users simultaneously communicate with the base station. The \(N\) subcarriers are evenly divided into \(Q\) \((Q > K)\) sub-channels, each of which has \(P = N/Q\) subcarriers. Each user only occupies one subchannel, and the \(q\) th subchannel is assigned to the \(k\) th user, whose subcarrier indices is defined as \(\{q, q+1, \cdots, (P-1)Q+q\}\) \((q = 0, 1, \cdots, Q-1)\). After the cyclic prefix (CP) is removed, the signal sample of the \(n\) th subcarrier of one OFDMA block at the base station (BS) can be described as

\[
y(n) = \sum_{k=0}^{K} y_k(n) + z(n) \quad (n = 0, 1, \cdots, N-1)
\]

where \(K\) denotes the number of all active users. \(z(n)\) is modeled as a white additive Gaussian noise with zero-mean and equal variance \(\sigma^2\). \(y_k(n)\) is the signal sample of the \(n\)th subcarrier from the \(k\) th user at the uplink receiver given by

\[
y_k(n) = \sum_{p=0}^{P-1} H_{k,p} X_{k,p} e^{j2\pi(p q + q_k)}
\]
where $H_{k,p}$ and $X_{k,p}$ denote the channel frequency response and the data symbol on the $p$ th subcarrier of the $k$ th user’s, respectively. $\xi_k = \Delta f_k / \Delta f$ is the normalized CFO of the $k$ th user. $\Delta f$ and $\Delta f_k$ are the sub-carrier spacing and the CFO between the $k$ th user and the BS, respectively. $| \Delta f_k |$ is assumed to be less than half of OFDMA subcarrier spacing, i.e., $| \xi_k | < 0.5$.

From (1), we can construct a $Q \times P$ matrix $Y$ by a data stacking technology. The matrix $Y$ has the following form

$$Y = \begin{bmatrix}
y(0) & y(1) & \cdots & y(P-1) \\
y(P) & y(P+1) & \cdots & y(2P-1) \\
\vdots & \vdots & \ddots & \vdots \\
y(N-P) & y(N-P+1) & \cdots & y(N-1) 
\end{bmatrix} = V_{Q \times K} S_{K \times P} + Z_{Q \times P} \quad (3)$$

where $S = C_{K \times P} B_{K \times P} W_{P \times P}$. The $k$ th row of $S$ is $[y_k(0), y_k(1), \cdots, y_k(P-1)]$, which denotes the received signals of the $k$ th user. The $k$ th column of $V$ is $v_k = [1, e^{j2\pi \theta_k}, e^{j4\pi \theta_k}, \cdots, e^{j2\pi(P-1)\theta_k}]^T$, where $\theta_k = (q + \xi_k) / Q$ is the effective CFO of the $k$ th user. $C = [c_1^T, c_2^T, \cdots, c_K^T]^T$ with $c_k = [1, e^{j2\pi \theta_k}, e^{j4\pi \theta_k}, \cdots, e^{j2\pi(P-1)\theta_k}]^T$, and $B = [b_1^T, b_2^T, \cdots, b_K^T]^T$ with $b_k = [H_{k,0} X_{k,0}, H_{k,1} X_{k,1}, \cdots, H_{k,P-1} X_{k,P-1}]^T$. $W$ is an IFFT matrix and $Z$ is the white Gaussian noise matrix.

Notice that the effective CFO has one important property. Different users have distinct effective CFOS. It can be shown that if one user occupies subchannel $q$, the range of its effective CFO is $((q-0.5)/Q, (q+0.5)/Q)$, since the range of $\xi_k$ is $(-0.5, 0.5)$. Because different users occupy different subchannels, their effective CFOS fall in nonoverlapping ranges.

Given the observed data matrix $Y$, the task is to estimate the effective CFOS $\theta_k (k = 1, 2, \cdots, K)$.

3. Algorithm formulation

Let $y_i$, $s_i$ and $z_i$ denote the $l$ th column of $Y$, $S$, and $Z$ ($l = 1, 2, \cdots, P$). Thus, we have the following equation

$$y_i = V s_i + z_i \quad (4)$$

where the $Q \times K$ Vandermonde matrix $V = [v_1, v_2, \cdots, v_K]$ with $v_k = [1, e^{j2\pi \theta_k}, \cdots, e^{j2\pi(P-1)\theta_k}]^T$ ($k = 1, 2, \cdots, K$).

We define a phase factor (CFO matrix) matrix $\Phi_Q$ as follows

$$\Phi_Q = \begin{bmatrix}
e^{j2\pi(1-Q)\theta_k} \\
e^{j2\pi(2-Q)\theta_k} \\
\vdots \\
e^{j2\pi(P-Q)\theta_k}
\end{bmatrix} = \begin{bmatrix}
e^{j\pi(1-Q)\theta_k} \\
e^{j\pi(2-Q)\theta_k} \\
\vdots \\
e^{j\pi(P-Q)\theta_k}
\end{bmatrix} \quad (5)$$

where $\theta_k$ is the effective CFO of the $k$ th user, which is given in (3).

In generally, $Q$ is an even number in the practical OFDMA systems. Using the above matrix $\Phi_Q$, (4) can be factored as
\[ y_i = V \Phi_0 \Phi_0^* s_i + z_i = A \Phi_0^* s_i + z_i \quad (6) \]

where \( A = V \Phi_0 = [a_1, \ldots, a_k] \), in which \( a_k = e^{j \pi (Q-1) \theta_k} v_k \) has the following form

\[ a_k = [e^{j \pi (Q-1) \theta_1}, \ldots, e^{j \pi \theta_1}, e^{j \pi (Q-1) \theta_2}, \ldots, e^{j \pi \theta_2}]^T \quad (7) \]

Notice that the column vector \( a_k \) contains the information of the CFO of \( k \)th user in the OFDMA uplink systems. Throughout the sequel, the vector \( a_k \) is referred to as the CFO steering vector and the matrix \( A \) is referred to as the CFO steering matrix.

The following developments in this paper rely on the centro-Hermitian property of the \( Q \times 1 \) CFO steering vector \( a_k \). The centro-Hermitian property of \( a_k \) is mathematically stated as

\[ a_k = \Pi_Q^* a_k^* \quad (8) \]

where the notation \((\cdot)^*\) denotes the complex conjugate operator. The matrix \( \Pi_Q \) is the \( Q \times Q \) exchange matrix with ones on its antidiagonal and zeros elsewhere.

Making use of the centro-Hermitian property of the CFO steering vector \( a_k \), we develop an invariance relationship satisfied by the real-valued CFO steering vector that involves only real-valued quantities. Firstly, we define the \( Q \times Q \) unitary matrix \( Q_Q \) as follows

\[ Q_Q = \frac{1}{\sqrt{2}} \begin{bmatrix} I_m & jI_m \\ \Pi_m & -j \Pi_m \end{bmatrix} \quad (9) \]

\( Q_Q^U \) is a sparse unitary matrix that transforms the \( Q \times 1 \) centro-Hermitian vector \( a_k \) into the \( Q \times 1 \) real-valued CFO steering vector \( d_k = Q_Q^U a_k \). The real-valued CFO steering vector \( d_k \) has the following form

\[ d_k = Q_Q^U a_k = \sqrt{2} \times \begin{bmatrix} \cos((Q-1)\pi \theta_1), \ldots, \cos(\pi \theta_1), -\sin((Q-1)\pi \theta_1), \ldots, -\sin(\pi \theta_1) \end{bmatrix}^T \quad (10) \]

The CFO steering vector \( a_k \) in (7) satisfies the invariance relation

\[ e^{j 2 \pi \theta} J_1 a_k = J_2 a_k \quad (11) \]

where \( J_1 \) and \( J_2 \) are the \( (Q-1) \times Q \) selection matrices

\[ J_1 = \begin{bmatrix} I_{Q-1}, 0_{(Q-1)1} \end{bmatrix} \quad J_2 = \begin{bmatrix} 0_{(Q-1)1}, I_{Q-1} \end{bmatrix} \quad (12) \]

where \( I_{Q-1} \) and \( 0_{(Q-1)1} \) present the \( (Q-1) \times (Q-1) \) identity matrix and \( (Q-1) \times 1 \) zero matrix, respectively.

Since \( Q_Q \) is unitary, it follows that \( e^{j 2 \pi \theta} J_1 Q_Q Q_Q^U a_k = J_2 Q_Q Q_Q^U a_k \), invoking the definition of \( d_k \) in (10), implies \( e^{j 2 \pi \theta} J_1 Q_Q d_k = J_2 Q_Q d_k \). Premultiplying both sides by \( Q_Q^{U*} \) yields the following invariance relationship
\[ e^{j2\pi\theta} Q^H_{0,-j} J_1 Q_d d_k = Q^H_{0,-j} J_2 Q_d d_k \]  
(13)

Note that \( J_1 \) and \( J_2 \) satisfy \( J_1 J_2 = J_2 J_1 \). As a consequence,

\[ Q^H_{0,-j} J_1 Q_d = Q^H_{0,-j} J_2 Q_d = Q^H_{0,-j} J_2 Q_d (Q^H_{0,-j} J_1 Q_d)^{-1} \]  
(14)

where we have exploited the fact that \( Q^H_{0,-j} Q_d = Q_d^H Q_d = I \) for any \( Q_d \).

Let \( K_1 = \text{Re}\{Q^H_{0,-j} J_1 Q_d\} \) and \( K_2 = \text{Im}\{Q^H_{0,-j} J_1 Q_d\} \), where \( \text{Re}\{\} \) and \( \text{Im}\{\} \) stand for the real and imaginary part of a complex matrix or vector, respectively. Therefore, (13) can be expressed as

\[ e^{j2\pi\theta} (K_1 - jK_2) d_k = e^{-j2\pi\theta} (K_1 + jK_2) d_k \]  
(15)

Rearranging (15), we have

\[ (e^{j2\pi\theta} - e^{-j2\pi\theta}) K d_k = j(e^{j2\pi\theta} + e^{-j2\pi\theta}) K d_k \]  
(16)

Invoking the definition of the tangent function yields the following relationship satisfied by \( d_k \)

\[ \tan(2\pi\theta) K d_k = K d_k \]  
(17)

For \( K < Q \) users, we define the \( Q \times K \) real-valued CFO steering matrix as \( D = [d_1, \cdots, d_K] \). The real-valued CFO steering vector relation in (17) can be translated into the real-valued CFO steering matrix relation

\[ K_1 D = K_2 D \]  
(18)

where \( Q_k = \text{diag}\{\tan(2\pi\theta_1), \cdots, \tan(2\pi\theta_K)\} \).

Assume that the eigenvalue decomposition (EVD) of the covariance matrix \( R = E\{x x^T\} \)

\[ (x = [\text{Re}\{Q^H_{0,-j} y\}, \text{Im}\{Q^H_{0,-j} y\}]) \] has the following expression

\[ R = \sum_{m=1}^{Q} \lambda_m u_m u_m^H = U_m \Lambda_m U_m^H + U_m \Lambda_m U_m^H \]  
(19)

where \( \lambda_1 \geq \cdots \geq \lambda_Q \geq \lambda_{Q+1} = \cdots = \lambda_Q \), \( U_m = [u_1, \cdots, u_m] \), \( U_m = [u_{m+1}, \cdots, u_Q] \).

Since \( \text{span}\{U_m\} = \text{span}\{D\} \), there must exist a unique nonsingular real-valued matrix \( T \) such that \( U_m = DT \). Substituting \( D = UT^{-1} \) into (18) yields

\[ K_1 U_m \Phi = K_2 U_m \]  
(20)

where \( \Phi = T^{-1} Q \). That is, \( \Phi \) and \( Q \) have the same eigenvalues \( \tan(2\pi\theta_k) (k = 1, 2, \cdots, K) \). Thus, CFOs can be obtained via the eigenvalues of \( \Phi \).

Unitary-ESPRIT algorithm is summarized as follows:

1. Form the data matrix \( Y \) according to (3) and compute the covariance matrix \( R = E\{x x^T\} \)

\[ (x = [\text{Re}\{Q^H_{0,-j} y\}, \text{Im}\{Q^H_{0,-j} y\}]) \].

2. Compute the EVD of \( R = \sum_{m=1}^{Q} \lambda_m u_m u_m^H \), where \( \lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_Q \).
(3) Estimate the signal subspace $U_s$ via $K$ “largest” eigenvectors of $R$.
(4) Calculate $\Phi$ as the solution to the matrix equation $K_s U_s \Phi = K_s U_s$.
(5) Compute $\gamma_k (k = 1, \cdots, K)$ as the eigenvalues of $\Phi$.
(6) Estimate the CFOs as $\theta_k = \frac{1}{\pi} \tan^{-1}(\gamma_k), (k = 1, \cdots, K)$.

4. Computational complexity analysis of the related methods

We briefly assess the computational complexity of the proposed method. We use $O(M)$ to represent the order of $M$ real multiplications.

In the proposed method, the major computational complexity exists in the construction and the eigendecomposition of the $Q \times Q$ real covariance matrix $R$, Moore-Penrose inverses of $(Q-1) \times K$ real-valued matrix $K_s U_s$, and the eigendecomposition of $K \times K$ real-valued matrix $(K_s U_s) (K_s U_s)^\dagger$. The construction of the $Q \times Q$ matrix $R$ needs to $O(6PQ^2)$ multiplications. When $Q$ is big, compared with the total complexity of the algorithm, the computation for this step is relatively small. The eigendecomposition of the matrices $R$ and $(K_s U_s) (K_s U_s)^\dagger$ requires $O(Q^3)$ and $O(K^3)$ multiplications, respectively. Since $Q > K$, the first eigendecomposition is the most expensive. The computation of $X^\dagger = X^\dagger (XX^\dagger)^{-1}$ (where $X = K_s U_s$) calls for about $O(2(Q-1)^2 K)$ multiplications (ignoring the inversion).

Table 1. Comparison of the computational complexity of the proposed algorithm with the ESPRIT algorithm.

<table>
<thead>
<tr>
<th>Algorithms</th>
<th>Unitary-ESPRIT</th>
<th>ESPRIT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Covariance matrix</td>
<td>$O(6PQ^2)$</td>
<td>$O(4PQ^2)$</td>
</tr>
<tr>
<td>Eigendecomposition</td>
<td>$O(Q^3) + O(K^3)$</td>
<td>$O(8Q^3) + O(8K^3)$</td>
</tr>
<tr>
<td>Moore-Penrose</td>
<td>$O(2(Q-1)^2 K)$</td>
<td>$O(8(Q-1)^2 K)$</td>
</tr>
</tbody>
</table>

Table 1 presents the computational complexity of the ESPRIT algorithm by Junghoon Lee [14] and the proposed Unitary-ESPRIT method, including the aforementioned three main steps.

5. Simulation results

In this section, we construct several simulations to demonstrate the effectiveness of the proposed algorithm.

The interleaved uplink OFDMA system with 512 subcarriers is considered in the simulation. The length of the cycle prefix (CP) is set to 64 to accommodate the channel delay. Each user transmits his data stream using QPSK signals. Rayleigh fading channel model is used to generate the channel taps, and exponentially decaying power delay profiles is selected. Assume that there are 6 taps, and channels of different users are statistically independent of each other. The normalized CFOs of all active users are generated as random variables that are uniformly distributed in $(-0.5, 0.5)$ with 500 random trial runs.

The root mean square error (RMSE) is chosen to evaluate the performance of the proposed algorithm. The RMSE is defined as

$$\frac{1}{K\Pi} \sum_{\rho=1}^{M} \sum_{k=1}^{K} \left( \hat{\theta}_{\rho,k} - \theta_{\rho,k} \right)^2,$$

(21)
where \( N \) is the total number of Monte Carlo trials, \( \hat{\xi}_{p,k} \) is the estimate of \( \xi_{p,k} \), and \( \rho \) is the index of the Monte Carlo trials.

First, we compare the estimation performance of the proposed Unitary-ESPRIT algorithm with that of the ESPRIT method. In this simulation, the total subcarriers are divided into \( Q = 16 \) subchannels, and the active user number is \( K = 4 \). Fig.1 depicts the normalized RMSE curves of the proposed algorithm and the ESPRIT method. As shown in Fig.1, the proposed algorithm can work well for different SNR. Furthermore, under the circumstances where the SNR is relatively low, the estimation accuracy of the proposed method obviously outperforms that of the ESPRIT method. Meanwhile, in this simulation, we compare the time consumption of the proposed method and the ESPRIT method when the SNR varies from 0 dB to 30 dB with 500 random trial runs. The total execution time of the proposed Unitary-ESPRIT method is 24.3750 s, while that of the ESPRIT method is 32.4380 s.

**Figure 1.** Performance comparison of ESPRIT and Unitary-ESPRIT algorithms.

In the second test example, we evaluate the performance of the proposed Unitary-ESPRIT algorithm for different subchannel number. Here, we consider the OFDMA uplink system has 8 and 16 subchannels, respectively. As can be seen from Fig.2, the performance for 16 subchannels is better
than that of 8 subchannels. From Fig.2, It is easy to know that, when the user number is fixed, increasing the number of subchannels can improve the performance of estimation.

In the third test example, we assess the performance of the proposed Unitary-ESPRIT algorithm for different number of users. Fig.3 shows the RMSE curves of the proposed algorithm for $K = 2, 4, 8$, respectively. We observe that the RMSE for $K = 2$ is the smallest compared with those of $K = 4, 8$. When the user number degrades, the estimation performance increases.

![Figure 3. Performance comparison of Unitary-ESPRIT algorithm for various $K$.](image)

6. Conclusions

In this paper, Unitary-ESPRIT algorithm is proposed for CFO estimation in interleaved OFDMA uplink systems. The proposed algorithm which has an ESPRIT structure is derived in terms of the real-valued computations except the data preprocessing. In the proposed method, the computational complexity is reduced significantly by the real-valued computation, and the improved CFO estimation accuracy is obtained by using the data stacking technique and the unitary transformation. Compared with other existed CFO estimation approaches, the proposed method has lower computational complexity, but it exhibits superior performance, such as smaller estimation error and better robustness to low SNR, etc.

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8. References